Homework 10 Extra Problems - Solutions

Problem 1

(a) If the variable t is changed to $2t$ in both places it appears, the surface doesn't change. Since no domain is given, we can assume that t ranges through all real numbers. So t and $2t$ range through the same domain, just at different "rates". The t-constant grid curves are the same (with different constants) and the s-constant grid curves are the same circles (with the "particle" travelling twice as fast around each circle).

(b) The surface is the paraboloid, since we are "stacking" circles of radius s and height s^2. Since the equation of the parabolic bowl is $y = x^2 + z^2$, another parametrisation of the bowl is

$$\mathbf{r}(u, v) = \langle u, u^2 + v^2, v \rangle.$$

(To get this, we can use the general form of a parametrisation of a function.)

(c) It is $y = x^2 + z^2$. Plugging either parametrisation into the quantity $y - x^2 - z^2$ gives 0.

Problem 2

We will use the earth's radius as

$$r = 6371\,\mathrm{km}\,.$$

The earth's radius varies, so you may have a slightly different value; that's ok (I've used the average radius, but I'll be interested if someone does sufficient research to find a more accurate radius for the earth at Vancouver). The latitude and longitude of Vancouver are

49:15:00N, 123:04:48W

We'll approximate these with

$$49\pi/180 \text{ and } 123\pi/180$$

If you did this more precisely, that's great. But I'm allowing you to round to the closest degree (although one degree difference in longitude can be more than 100 km near the equator!). Since geographic latitude measures the angle up from the equator, and the longitude of Vancouver is given as *west* of Greenwich we use

$$\theta = -123\pi/180, \quad \phi = \pi/2 - 49\pi/180 = 41\pi/180$$

in the usual spherical coordinates. Plugging into the spherical coordinate expressions

$$x = r \sin\phi \cos\theta, \quad y = r \sin\phi \sin\theta, \quad z = r \cos\phi$$

we obtain

$$x = -2276\,\mathrm{km}, \quad y = -3505\,\mathrm{km} \quad z = -4808\,\mathrm{km}$$

We can catch some potential computational mistakes by checking that $x^2 + y^2 + z^2$ is approximately 6371.