# Reviewing for Math 317

## Computational Review

Old exam problems are really great for computational review. I recommend April 2009 as a very standard sort of Math 317 exam with a range of problems that test your ability to use the main theorems and do the standard computations. It also has some true-and-false questions. Once you've done the last homework and some review problems from the book, and possibly other exam problems from the archive as review, you can test your readiness for our exam by taking the April 2009 exam. Solutions are available in the packet which the Math Club produces (www.ubcmathclub.org). The math club booklet also has other exams with solutions.

More **true-and-false** can be found in April 2008, April 2007, April 2006, April 2005, and possibly others. These are all available online from the mathematics department website:

**http://www.math.ubc.ca/Ugrad/pastExams/index.shtml**

## Conceptual Review

These are open-ended questions, meant to guide your conceptual studying, which is more difficult than computational studying. These require creativity based on real conceptual understanding. You should consider this as a guide – by immitating this guide in style, you can make conceptual study guides for your other courses.

I am always available by email and frequently in person, and I welcome questions and discussion of these.

### Some of the big facts

1. Explain why the integral on a closed loop of a conservative vector field is zero by ...
   (a) ... using the Fundamental Theorem of Line Integrals.
   (b) ... using Green's Theorem in 2D.
   (c) ... using Stokes' Theorem in 3D.
2. Explain why the integral on a closed surface of curl field in 3D (i.e. a vector field which is the curl of another vector field) is zero by ...
   (a) ... using Stokes' Theorem.
   (b) ... using the Divergence Theorem.
3. We have fundamental derivatives grad, div and curl.
   (a) Which of them can be taken in sequence? I.e. list all of the ways it makes sense (is well defined) to do one of these followed by another one.
   (b) Which of them can be taken in sequence and results always in 0? (Hint: there are two such.)
4. How can you tell when a field is conservative?
5. Make sure to collect the main theorems from the course into a list.

6. Explain the relationship between curl and circulation.

7. Explain the relationship between divergence and flux.

**Some of the big examples**

1. Consider the vector field $\mathbf{F} = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$. This field has been an important example throughout the semester. For this field ...

   (a) Find div $\mathbf{F}$.

   (b) Find the domain of $\mathbf{F}$.

   (c) Find curl $\mathbf{F}$.

   (d) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ around the unit circle $C$ directly.

   (e) Is this field conservative? Why?

   (f) Does The Fundamental Theorem of Line Integrals apply to this field? Why?

   (g) Does Green's Theorem apply to this field? Why/when?

   (h) Find a path in this field for which $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$. What about a non-trivial closed loop?

   (i) If you restrict the domain of this field to the part of the plane satisfying $x > 1$, is it a conservative field?

   (j) (Harder / further exploration:) If the last question has answer "yes", then what is the potential function? Why isn't this a perfectly good potential function on the full domain of the field? If it is, why do you get a non-zero line integral on the unit circle?

2. Consider the vector field $\mathbf{F} = \langle \frac{x}{(x^2+y^2+z^2)^{3/2}}, \frac{y}{(x^2+y^2+z^2)^{3/2}}, \frac{z}{(x^2+y^2+z^2)^{3/2}} \rangle$. This field is an important example also. For this field ...

   (a) Find div $\mathbf{F}$.

   (b) Find the domain of $\mathbf{F}$.

   (c) Find curl $\mathbf{F}$.

   (d) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ around the unit circle $C$ directly.

   (e) Compute the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ on the unit sphere $S$ directly.

   (f) Is this field conservative? Why?

   (g) Is this field a curl field? Why?

   (h) Does Stokes' Theorem apply to this field? Why?

   (i) Does the Divergence Theorem apply to this field? Why?

   (j) Find a surface in this field for which $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$.

### Creating your own examples

The ability to create your own examples of concepts in class requires understanding. What follows is a list of things you could try to create an example of. Some of the following are quick, some are harder. Some you can do by trial and error, but you will learn more if you find ways to do them through understanding. Can you create a whole family of examples? Can you come up with a method for generating tons of examples? Be warned: a few of them are impossible. In this case, explain why they are impossible. Also, this list is by no means exhaustive. Add your own exercises to the list.

1. A conservative vector field (2D and 3D).
2. A non-conservative vector field (2D and 3D).
3. A conservative vector field with non-trivial curl.
4. A 2D vector field with a non-zero line integral around the unit circle.
5. A 2D vector field with a zero line integral around the unit circle.
6. A 2D vector field with a zero line integral around one loop and a non-zero line integral around a different loop.
7. A 3D vector field which is not the curl of another field.
8. A 3D vector field which is the curl of another field.
9. A 3D vector field with a zero surface integral on the unit sphere.
10. A 3D vector field with a non-zero surface integral on the unit sphere.
11. A 3D vector field with non-zero divergence everywhere.
12. A 3D vector field with zero divergence everywhere.
13. A 3D vector field with constant non-zero divergence.
14. A 3D vector field with constant non-zero curl.
15. A disconnected domain.
16. A connected but not simply-connected domain.
17. A simply-connected domain.
18. A vector field whose domain is not simply-connected.
19. A vector field whose domain is not connected.
20. A conservative vector field with a closed loop on which the line integral is non-zero.
21. An incompressible vector field on which the flux integral on some closed surface is non-zero.
22. An irrotational field.

23. A 2D field for which the line integral inside any closed loop is equal to the area contained in the loop.

24. A 3D field for which the surface integral on any closed surface is equal to the volume contained in the surface.

25. A closed surface.

26. A non-closed surface.

27. A closed loop.

28. A non-closed curve.

29. A surface with no boundary.

30. A surface with a boundary which is not a simple curve.

31. A surface with a boundary which is a curve with endpoints.

32. An orientable surface.

33. A non-orientable surface.

34. A non-orientable curve.

35. A volume with no boundary.

36. A volume whose boundary has no boundary.

37. A volume whose boundary has boundary.

38. Two parametrisations of the same surface that give opposite orientations.

39. Two parametrisations of the same surface that give different grid curves.

40. Two parametrisations of the same curve that give different curvature.

41. Two parametrisations of the same curve that give different arclength.

42. Two parametrisations of the same curve that give different a different acceleration.

43. A really really really ugly parametrisation of the cylinder. (How ugly can you make it without messing up the fact that it's a cylinder?)

44. A curve whose osculating circle is always the same radius.

45. A curve of constant curvature. Can you give another?

46. A curve whose curvature is always increasing in one direction.

47. A curve with zero curvature.

48. A curve with negative arclength.

49. A surface with surface area 10.

50. A surface integral $\iint_S f dS$ that comes out to -1.
51. A parametrised curve with zero acceleration.
52. A parametrised curve with no tangential acceleration but non-zero normal acceleration.
53. A parametrised curve of constant non-zero acceleration.
54. Re-parametrise the last curve so that it has zero acceleration (constantly).
55. Re-parametrise the circle so that it has zero acceleration (constantly).
56. Make up your own. I'm just sitting here typing these at a rate of one per second...

**Explaining general methods and making your own problems**

Imagine you are teaching this course. How would you explain, in general, how to attack integrals? Explain, in general terms, how to use the handout from Dec 1st (available online) to figure out which theorems could be applied to compute a given integral.

Then, imagine you are writing the final exam! Design the following exam problems. To make one of these, you might look back at problems we've done and see what properties the fields or curves or surfaces had that made it possible or advantageous to use a particular approach (a particular big theorem). Then try to make up a new example with the right properties. This involves some trial-and-error, but try to look for general principles as you try things. I f you can *design* exam problems, you will never be stumped by them!

1. a line integral that is hard to do directly but you can use the Fundamental Theorem of Line Integrals to do easily.
2. a line integral that is hard to do directly and can't be done with the Fundamental Theorem of Line Integrals (say why), but can be done fairly easily with Green's Theorem.
3. a surface integral that is hard to do directly but can be done fairly easily with Stokes' Theorem.
4. a surface integral that is hard to do directly but can be done fairly easily using the Divergence Theorem.
5. a non-closed loop line integral that is hard to do directly but can be done fairly easily with Green's Theorem.
6. a non-closed surface integral that is hard to do directly but can be done fairly easily using the Divergence Theorem.
7. other variations?