Midterm #2 REVIEW PROBLEMS

Math 317 November 5, 2010

Name ______________________________ **Student Number** ______________________________

No books, notes or calculators are allowed. Unless otherwise stated, to receive full credit you must provide clear, tidy, understandable justification for your solutions.

Practice Problems

A. Find a potential function for the vector field

$$\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$$

Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from $(1, 0, -2)$ to $(4, 6, 3)$.

B. Evaluate the integral

$$\int_C (y + e^{\sqrt{x}})dx + (2x + cos(y^2))dy$$

where C is the boundary (oriented in the counterclockwise direction) of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

C. Suppose we know that the vector field

$$\mathbf{F} = Ax^3y^2z\mathbf{i} + (z^3 + Bx^4yz)\mathbf{j} + (3yz^2 - x^4y^2)\mathbf{k}$$

is conservative. Find the values of A and B. Find a scalar function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

D. Let C be the oriented curve in the xy plane from $(0, 0)$ to $(\pi, 0)$ given by $y = \sin(x)$. Let

$$\mathbf{F} = (x + y)\mathbf{i} + (2x - e^{\sqrt{y}})\mathbf{j}$$

Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ (hint: do not try to evaluate the integral directly).

E. Find the work done by the force field

$$\mathbf{F}(x, y) = x\mathbf{i} + (y + 2)\mathbf{j}$$

in moving an object along an arch of the cycloid

$$\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}, \quad 0 \le t \le 2\pi$$

F. Construct an example of a 2D vector field... (each of the following is a separate question; there's no vector field that satisfies all of these at once!)

(a) which is conservative.

(b) which is not conservative.

(c) which has curl equal to x everywhere.

(d) for which $\int_C \mathbf{F} \cdot d\mathbf{r} = 1$ for C the unit circle.

(e) which has potential function $f(x, y) = 2y^2x$.

(f) whose domain is not connected.

(g) whose domain is not simply connected.

(h) that you particularly like (why?).

In each case, show that your answer is correct. In each case, explain whether the answer is unique. If it is not unique, come up with a second, different example.

G. Don't forget to do some vector field graph matching or graphing (handout / textbook).

H. Say whether the following statements are true (**T**) or false (**F**). You may assume that all functions are defined everywhere and have continuous derivatives of all orders everywhere. You may also assume all curves are smooth and all regions are open. Give explanations for your answer and if it is false, correct it to make a true statement.

T F In a simply connected region, $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of C.

T F If $\nabla f = 0$, then f is a constant function.

T F If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed curve C, then $\mathbf{F}$ is conservative.

T F If $\mathbf{F} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ satisfies $Q_y = P_x$, then $\mathbf{F}$ is conservative.

T F Let $A(t)$ be the area swept out by the trajectory of a planet from time t_0 to t. Then $\frac{dA}{dt}$ is constant.

T F If the domain of a 2D vector field $\mathbf{F}$ is connected, and the curl of $\mathbf{F}$ is zero everywhere, then $\mathbf{F}$ is conservative.

T F If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed curve C in the domain of $\mathbf{F}$, then there is a function f such that $\mathbf{F} = \nabla f$.

T F If you go for a run around the block, returning to where you began, and you have a headwind the entire way, then the total work done by the wind on you during your run is positive.

T F If a vector field $\mathbf{F}$ is conservative, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed loops C.

T F The vector field $\mathbf{F} = \langle 1, 1 \rangle$ is conservative.

T F If line integrals $\int_C \mathbf{F} \cdot d\mathbf{r}$ are path independent for a vector field $\mathbf{F}$, then the vector field is conservative.

T F If a vector field $\mathbf{F}$ is parallel to $\mathbf{T}$ everywhere on a curve C, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$.

T F If $\mathbf{F}$ is a conservative force field, and a particle moves in the field according to the force, its potential energy could increase, decrease, or stay the same.

T F The flow lines of a velocity field satisfy $\mathbf{r}(t) = \mathbf{F}(t)$.

T F If C is a positively oriented curve and $-C$ is the curve traversed in the opposite orientation, then $\int_C f ds = -\int_{-C} f ds$.

T F A planet orbiting the sun in a non-circular elliptical orbit has constant acceleration.