SAMPLE OLD MIDTERM QUESTIONS Math 317

Name ______________________________ **Student Number** ______________________________

No books, notes or calculators are allowed.

The following formulas may be useful:

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \quad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} \quad \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}$$

Problem A Sketch a graph of the space curve $\mathbf{r}(t) = \langle \cos(t^3+t), \sin(t^3+t), 5\rangle$. Indicate the following things on your graph:

- the directional arrow for the curve (direction of increasing t)
- the point $P = (1, 0, 5)$
- the point Q corresponding to $t^3 + t = \pi$
- the vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ at Q.

Problem B Say whether the following statements are true (**T**) or false (**F**). You may assume that all functions and vector fields are defined everywhere and have derivatives of all orders everywhere. You do not need to give reasons; this problem will be graded by answer only. **You will get +1 for each correct answer, -1 for each wrong answer, and 0 for each no answer.**

1. The curve with vector equation $\mathbf{r}(t) = \langle t^3, 2t^3, 3t^3\rangle$ is a line.
2. If $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are vector valued functions, then
$$\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}'(t).$$
3. The circle has constant curvature.
4. The integral of a vector function is obtained by integrating each component function.
5. If a vector function $\mathbf{r}(t)$ is such that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are always perpendicular, then $\mathbf{r}(t)$ is constant.

For good reviewing, give reasons for your choices above, and if you said false, can you correct the statement to make it true?

Problem C Consider the curve with the parameterization

$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$$

for $-\infty < t < \infty$.

1. Compute the curvature $\kappa(t)$ as a function of t.
2. What is the limit of $\kappa(t)$ as t approaches infinity? What is the limit of $\kappa(t)$ as t approaches minus infinity?
3. For what value of t is $\kappa(t)$ maximum?

Problem D

1. Find the length of the curve
$$\mathbf{r}(t) = e^t\mathbf{i} + e^t \sin t\mathbf{j} + e^t \cos t\mathbf{k}$$
from $t = 0$ to $t = T$ where T is any positive number.

2. Reparameterize $\mathbf{r}(t)$ with respect to arclength measured from the point where $t = 0$ in the direction of increasing t.

Problem E
Suppose that a particle of mass m with position vector $\mathbf{r}(t)$ is acted on by a central force

$$\mathbf{F} = f(r)\mathbf{r}, \quad \text{where } r = |\mathbf{r}|.$$

Define the *potential energy* $U(r)$ to be the antiderivative:

$$U(r) = -\int r f(r)\, dr$$

and define the *kinetic energy* $K(t)$ to be

$$K = \frac{1}{2} m |\mathbf{v}|^2.$$

Show that the total energy $E = K + U$ is conserved, i.e. show that $K + U$ is a constant with respect to t. (**Hint:** In order to compute dr/dt, write r as $\sqrt{\mathbf{r} \cdot \mathbf{r}}$.)

Problem F Select the correct answer; you do not need to show work; **no partial credit will be given**.
Let $\mathbf{r}(t)$ be a vector valued function. Let $\mathbf{r}'$, $\mathbf{r}''$, and $\mathbf{r}'''$ denote $\frac{d\mathbf{r}}{dt}$, $\frac{d^2\mathbf{r}}{dt^2}$, and $\frac{d^3\mathbf{r}}{dt^3}$ respectively.
Then $\frac{d}{dt}\left[(\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}''\right]$ is given by

1. $(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''$
2. $(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r} + (\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}'''$
3. $\mathbf{r} \cdot (\mathbf{r}' \times \mathbf{r}''')$
4. $\mathbf{0}$
5. None of the above.

Problem G Suppose that the position vector $\mathbf{r}(t)$ of an object in motion is in the same direction as the acceleration vector $\mathbf{a}(t)$, in other words, there is a function $f(t)$ such that

$$\mathbf{a}(t) = f(t)\mathbf{r}(t).$$

Let $\mathbf{h}(t)$ be the cross product of the position vector and the velocity:

$$\mathbf{h}(t) = \mathbf{r}(t) \times \mathbf{v}(t).$$

Show that $\mathbf{h}(t)$ is a constant vector.