4. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \cos xz & -\sin xy \end{vmatrix} = (-x\cos xy + x\sin xz)\,\mathbf{i} - (-y\cos xy - 0)\,\mathbf{j} + (-z\sin xz - 0)\,\mathbf{k}$

$= x(\sin xz - \cos xy)\,\mathbf{i} + y\cos xy\,\mathbf{j} - z\sin xz\,\mathbf{k}$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial}{\partial x}(0) + \dfrac{\partial}{\partial y}(\cos xz) + \dfrac{\partial}{\partial z}(-\sin xy) = 0 + 0 + 0 = 0$

8. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^x & e^{xy} & e^{xyz} \end{vmatrix} = (xze^{xyz} - 0)\,\mathbf{i} - (yze^{xyz} - 0)\,\mathbf{j} + (ye^{xy} - 0)\,\mathbf{k}$

$= \langle xze^{xyz}, -yze^{xyz}, ye^{xy}\rangle$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \dfrac{\partial}{\partial x}(e^x) + \dfrac{\partial}{\partial y}(e^{xy}) + \dfrac{\partial}{\partial z}(e^{xyz}) = e^x + xe^{xy} + xye^{xyz}$

9. If the vector field is $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$, then we know $R = 0$. In addition, the x-component of each vector of $\mathbf{F}$ is 0, so $P = 0$, hence $\dfrac{\partial P}{\partial x} = \dfrac{\partial P}{\partial y} = \dfrac{\partial P}{\partial z} = \dfrac{\partial R}{\partial x} = \dfrac{\partial R}{\partial y} = \dfrac{\partial R}{\partial z} = 0$. Q decreases as y increases, so $\dfrac{\partial Q}{\partial y} < 0$, but Q doesn't change in the x- or z-directions, so $\dfrac{\partial Q}{\partial x} = \dfrac{\partial Q}{\partial z} = 0$.

(a) $\operatorname{div}\mathbf{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z} = 0 + \dfrac{\partial Q}{\partial y} + 0 < 0$

(b) $\operatorname{curl}\mathbf{F} = \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x}\right)\mathbf{j} + \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right)\mathbf{k} = (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = \mathbf{0}$

10. If the vector field is $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$, then we know $R = 0$. In addition, P and Q don't vary in the z-direction, so $\dfrac{\partial R}{\partial x} = \dfrac{\partial R}{\partial y} = \dfrac{\partial R}{\partial z} = \dfrac{\partial P}{\partial z} = \dfrac{\partial Q}{\partial z} = 0$. As x increases, the x-component of each vector of $\mathbf{F}$ increases while the y-component remains constant, so $\dfrac{\partial P}{\partial x} > 0$ and $\dfrac{\partial Q}{\partial x} = 0$. Similarly, as y increases, the y-component of each vector increases while the x-component remains constant, so $\dfrac{\partial Q}{\partial y} > 0$ and $\dfrac{\partial P}{\partial y} = 0$.

(a) $\operatorname{div}\mathbf{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + 0 > 0$

(b) $\operatorname{curl}\mathbf{F} = \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x}\right)\mathbf{j} + \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right)\mathbf{k} = (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = \mathbf{0}$

11. If the vector field is $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$, then we know $R = 0$. In addition, the y-component of each vector of $\mathbf{F}$ is 0, so $Q = 0$, hence $\frac{\partial Q}{\partial x} = \frac{\partial Q}{\partial y} = \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial x} = \frac{\partial R}{\partial y} = \frac{\partial R}{\partial z} = 0$. P increases as y increases, so $\frac{\partial P}{\partial y} > 0$, but P doesn't change in the x- or z-directions, so $\frac{\partial P}{\partial x} = \frac{\partial P}{\partial z} = 0$.

(a) $\operatorname{div} \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 0 + 0 + 0 = 0$

(b) $\operatorname{curl} \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k} = (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + \left(0 - \frac{\partial P}{\partial y}\right)\mathbf{k} = -\frac{\partial P}{\partial y}\mathbf{k}$

Since $\frac{\partial P}{\partial y} > 0$, $-\frac{\partial P}{\partial y}\mathbf{k}$ is a vector pointing in the negative z-direction.

12. (a) $\operatorname{curl} f = \nabla \times f$ is meaningless because f is a scalar field.

(b) $\operatorname{grad} f$ is a vector field.

(c) $\operatorname{div} \mathbf{F}$ is a scalar field.

(d) $\operatorname{curl}\,(\operatorname{grad} f)$ is a vector field.

(e) $\operatorname{grad} \mathbf{F}$ is meaningless because $\mathbf{F}$ is not a scalar field.

(f) $\operatorname{grad}(\operatorname{div} \mathbf{F})$ is a vector field.

(g) $\operatorname{div}(\operatorname{grad} f)$ is a scalar field.

(h) $\operatorname{grad}(\operatorname{div} f)$ is meaningless because f is a scalar field.

(i) $\operatorname{curl}(\operatorname{curl} \mathbf{F})$ is a vector field.

(j) $\operatorname{div}(\operatorname{div} \mathbf{F})$ is meaningless because $\operatorname{div} \mathbf{F}$ is a scalar field.

(k) $(\operatorname{grad} f) \times (\operatorname{div} \mathbf{F})$ is meaningless because $\operatorname{div} \mathbf{F}$ is a scalar field.

(l) $\operatorname{div}(\operatorname{curl}(\operatorname{grad} f))$ is a scalar field.

18. $\operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\cos xy & x\cos xy & -\sin z \end{vmatrix}$

$$= (0-0)\,\mathbf{i} - (0-0)\,\mathbf{j} + [(-xy\sin xy + \cos xy) - (-xy\sin xy + \cos xy)]\,\mathbf{k} = \mathbf{0}$$

$\mathbf{F}$ is defined on all of $\mathbb{R}^3$, and the partial derivatives of the component functions are continuous, so $\mathbf{F}$ is conservative. Thus there exists a function f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = y\cos xy$ implies $f(x,y,z) = \sin xy + g(y,z) \Rightarrow$ $f_y(x,y,z) = x\cos xy + g_y(y,z)$. But $f_y(x,y,z) = x\cos xy$, so $g(y,z) = h(z)$ and $f(x,y,z) = \sin xy + h(z)$. Thus $f_z(x,y,z) = h'(z)$ but $f_z(x,y,z) = -\sin z$ so $h(z) = \cos z + K$ and a potential function for $\mathbf{F}$ is $f(x,y,z) = \sin xy + \cos z + K$.

20. No. Assume there is such a $\mathbf{G}$. Then $\operatorname{div}(\operatorname{curl}\mathbf{G}) = yz - 2yz + 2yz = yz \neq \mathbf{0}$ which contradicts Theorem 11.

22. $\operatorname{div}\mathbf{F} = \dfrac{\partial(f(y,z))}{\partial x} + \dfrac{\partial(g(x,z))}{\partial y} + \dfrac{\partial(h(x,y))}{\partial z} = \mathbf{0}$ so $\mathbf{F}$ is incompressible.

For Exercises 23–29, let $\mathbf{F}(x,y,z) = P_1\,\mathbf{i} + Q_1\,\mathbf{j} + R_1\,\mathbf{k}$ and $\mathbf{G}(x,y,z) = P_2\,\mathbf{i} + Q_2\,\mathbf{j} + R_2\,\mathbf{k}$.