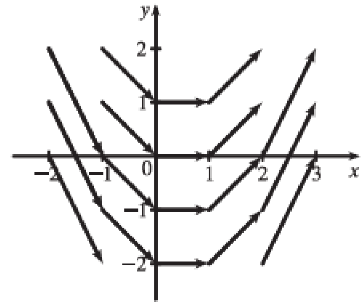
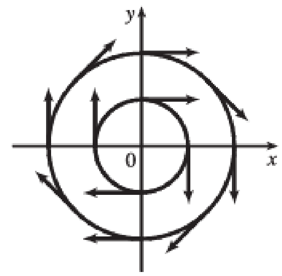
2. $\mathbf{F}(x, y) = \mathbf{i} + x\,\mathbf{j}$

The length of the vector $\mathbf{i} + x\,\mathbf{j}$ is $\sqrt{1 + x^2}$. Vectors are tangent to parabolas opening about the y-axis.



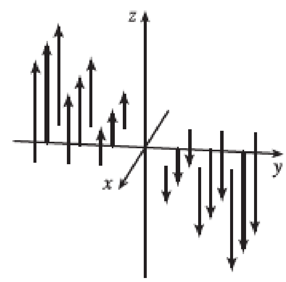
6. $\mathbf{F}(x, y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2 + y^2}}$

All the vectors $\mathbf{F}(x, y)$ are unit vectors tangent to circles centered at the origin with radius $\sqrt{x^2 + y^2}$.



8. $\mathbf{F}(x, y, z) = -y\,\mathbf{k}$

At each point (x, y, z), $\mathbf{F}(x, y, z)$ is a vector of length $|y|$. For $y > 0$, all point in the direction of the negative z-axis, while for $y < 0$, all are in the direction of the positive z-axis. In each plane $y = k$, all the vectors are identical.



11. $\mathbf{F}(x, y) = \langle y, x \rangle$ corresponds to graph II. In the first quadrant all the vectors have positive x- and y-components, in the second quadrant all vectors have positive x-components and negative y-components, in the third quadrant all vectors have negative x- and y-components, and in the fourth quadrant all vectors have negative x-components and positive y-components. In addition, the vectors get shorter as we approach the origin.

12. $\mathbf{F}(x, y) = \langle 1, \sin y \rangle$ corresponds to graph IV since the x-component of each vector is constant, the vectors are independent of x (vectors along horizontal lines are identical), and the vector field appears to repeat the same pattern vertically.

13. $\mathbf{F}(x, y) = \langle x - 2, x + 1 \rangle$ corresponds to graph I since the vectors are independent of y (vectors along vertical lines are identical) and, as we move to the right, both the x- and the y-components get larger.

14. $\mathbf{F}(x, y) = \langle y, 1/x \rangle$ corresponds to graph III. As in Exercise 11, all the vectors in the first quadrant have positive x- and y-components, in the second quadrant all vectors have positive x-components and negative y-components, in the third quadrant all vectors have negative x- and y-components, and in the fourth quadrant all vectors have negative x-components and positive y-components. Also, the vectors become longer as we approach the y-axis.

15. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph IV, since all vectors have identical length and direction.

16. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph I, since the horizontal vector components remain constant, but the vectors above the xy-plane point generally upward while the vectors below the xy-plane point generally downward.

17. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph III; the projection of each vector onto the xy-plane is $x\,\mathbf{i} + y\,\mathbf{j}$, which points away from the origin, and the vectors point generally upward because their z-components are all 3.

18. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph II; each vector $\mathbf{F}(x, y, z)$ has the same length and direction as the position vector of the point (x, y, z), and therefore the vectors all point directly away from the origin.

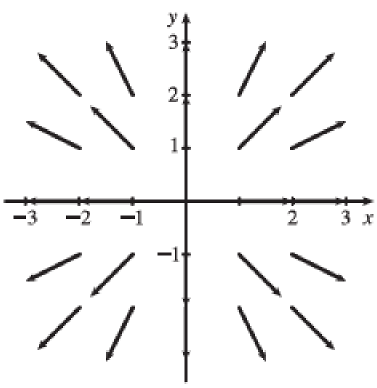
22. $f(x, y) = \tan(3x - 4y) \quad \Rightarrow$

$$\nabla f(x, y) = f_x(x, y)\,\mathbf{i} + f_y(x, y)\,\mathbf{j} = \left[\sec^2(3x - 4y) \cdot 3\right]\mathbf{i} + \left[\sec^2(3x - 4y) \cdot (-4)\right]\mathbf{j}$$
$$= 3\sec^2(3x - 4y)\,\mathbf{i} - 4\sec^2(3x - 4y)\,\mathbf{j}$$

26. $f(x, y) = \sqrt{x^2 + y^2} \quad \Rightarrow$

$$\nabla f(x, y) = \tfrac{1}{2}(x^2 + y^2)^{-1/2}(2x)\,\mathbf{i} + \tfrac{1}{2}(x^2 + y^2)^{-1/2}(2y)\,\mathbf{j}$$
$$= \frac{x}{\sqrt{x^2 + y^2}}\,\mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}}\,\mathbf{j} \text{ or } \frac{1}{\sqrt{x^2 + y^2}}\,(x\,\mathbf{i} + y\,\mathbf{j})\,.$$

$\nabla f(x, y)$ is not defined at the origin, but elsewhere all vectors have length 1 and point away from the origin.



29. $f(x, y) = x^2 + y^2 \quad \Rightarrow \quad \nabla f(x, y) = 2x\,\mathbf{i} + 2y\,\mathbf{j}$. Thus, each vector $\nabla f(x, y)$ has the same direction and twice the length of the position vector of the point (x, y), so the vectors all point directly away from the origin and their lengths increase as we move away from the origin. Hence, ∇f is graph II.

30. $f(x, y) = x(x + y) = x^2 + xy \quad \Rightarrow \quad \nabla f(x, y) = (2x + y)\,\mathbf{i} + x\,\mathbf{j}$. The y-component of each vector is x, so the vectors point upward in quadrants I and IV and downward in quadrants II and III. Also, the x-component of each vector is 0 along the line $y = -2x$ so the vectors are vertical there. Thus, ∇f is graph IV.

31. $f(x,y) = (x+y)^2 \quad\Rightarrow\quad \nabla f(x,y) = 2(x+y)\,\mathbf{i} + 2(x+y)\,\mathbf{j}$. The x- and y-components of each vector are equal, so all vectors are parallel to the line $y = x$. The vectors are $\mathbf{0}$ along the line $y = -x$ and their length increases as the distance from this line increases. Thus, ∇f is graph II.

32. $f(x,y) = \sin\sqrt{x^2+y^2} \quad\Rightarrow$

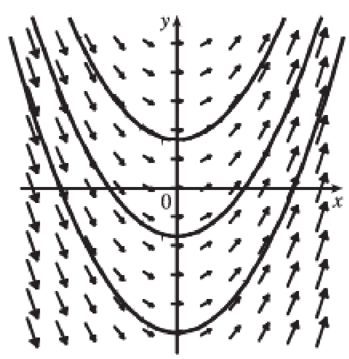
$$\nabla f(x,y) = \left[\cos\sqrt{x^2+y^2}\cdot\tfrac{1}{2}(x^2+y^2)^{-1/2}(2x)\right]\mathbf{i} + \left[\cos\sqrt{x^2+y^2}\cdot\tfrac{1}{2}(x^2+y^2)^{-1/2}(2y)\right]\mathbf{j}$$

$$= \frac{\cos\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,x\,\mathbf{i} + \frac{\cos\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,y\,\mathbf{j} \text{ or } \frac{\cos\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,(x\,\mathbf{i} + y\,\mathbf{j})$$

Thus each vector is a scalar multiple of its position vector, so the vectors point toward or away from the origin with length that changes in a periodic fashion as we move away from the origin. ∇f is graph I.

34. At $t = 1$ the particle is at $(1,3)$ so its velocity is $\mathbf{F}(1,3) = \langle 1,-1\rangle$. After 0.05 units of time, the particle's change in location should be approximately $0.05\,\mathbf{F}(1,3) = 0.05\,\langle 1,-1\rangle = \langle 0.05,-0.05\rangle$, so the particle should be approximately at the point $(1.05, 2.95)$.

36. (a) We sketch the vector field $\mathbf{F}(x,y) = \mathbf{i} + x\,\mathbf{j}$ along with several approximate flow lines. The flow lines appear to be parabolas.



(b) If $x = x(t)$ and $y = y(t)$ are parametric equations of a flow line, then the velocity vector of the flow line at the point (x,y) is $x'(t)\,\mathbf{i} + y'(t)\,\mathbf{j}$. Since the velocity vectors coincide with the vectors in the vector field, we have $x'(t)\,\mathbf{i} + y'(t)\,\mathbf{j} = \mathbf{i} + x\,\mathbf{j} \quad\Rightarrow\quad \dfrac{dx}{dt} = 1,\ \dfrac{dy}{dt} = x$. Thus $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{x}{1} = x$.

(c) From part (b), $dy/dx = x$. Integrating, we have $y = \frac{1}{2}x^2 + c$. Since the particle starts at the origin, we know $(0,0)$ is on the curve, so $0 = 0 + c \quad\Rightarrow\quad c = 0$ and the path the particle follows is $y = \frac{1}{2}x^2$.