23. $\operatorname{div}(\mathbf{F}+\mathbf{G}) = \operatorname{div}\langle P_1+P_2, Q_1+Q_2, R_1+R_2\rangle = \dfrac{\partial(P_1+P_2)}{\partial x} + \dfrac{\partial(Q_1+Q_2)}{\partial y} + \dfrac{\partial(R_1+R_2)}{\partial z}$

$$= \frac{\partial P_1}{\partial x} + \frac{\partial P_2}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial Q_2}{\partial y} + \frac{\partial R_1}{\partial z} + \frac{\partial R_2}{\partial z} = \left(\frac{\partial P_1}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial R_1}{\partial z}\right) + \left(\frac{\partial P_2}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{\partial R_2}{\partial z}\right)$$

$$= \operatorname{div}\langle P_1, Q_1, R_1\rangle + \operatorname{div}\langle P_2, Q_2, R_2\rangle = \operatorname{div}\mathbf{F} + \operatorname{div}\mathbf{G}$$

24. $\operatorname{curl}\mathbf{F} + \operatorname{curl}\mathbf{G} = \left[\left(\dfrac{\partial R_1}{\partial y} - \dfrac{\partial Q_1}{\partial z}\right)\mathbf{i} + \left(\dfrac{\partial P_1}{\partial z} - \dfrac{\partial R_1}{\partial x}\right)\mathbf{j} + \left(\dfrac{\partial Q_1}{\partial x} - \dfrac{\partial P_1}{\partial y}\right)\mathbf{k}\right]$

$$+ \left[\left(\frac{\partial R_2}{\partial y} - \frac{\partial Q_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P_2}{\partial z} - \frac{\partial R_2}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q_2}{\partial x} - \frac{\partial P_2}{\partial y}\right)\mathbf{k}\right]$$

$$= \left[\frac{\partial(R_1+R_2)}{\partial y} - \frac{\partial(Q_1+Q_2)}{\partial z}\right]\mathbf{i} + \left[\frac{\partial(P_1+P_2)}{\partial z} - \frac{\partial(R_1+R_2)}{\partial x}\right]\mathbf{j}$$

$$+ \left[\frac{\partial(Q_1+Q_2)}{\partial x} - \frac{\partial(P_1+P_2)}{\partial y}\right]\mathbf{k} = \operatorname{curl}(\mathbf{F}+\mathbf{G})$$

25. $\operatorname{div}(f\mathbf{F}) = \operatorname{div}(f\,\langle P_1, Q_1, R_1\rangle) = \operatorname{div}\langle fP_1, fQ_1, fR_1\rangle = \dfrac{\partial(fP_1)}{\partial x} + \dfrac{\partial(fQ_1)}{\partial y} + \dfrac{\partial(fR_1)}{\partial z}$

$$= \left(f\,\frac{\partial P_1}{\partial x} + P_1\,\frac{\partial f}{\partial x}\right) + \left(f\,\frac{\partial Q_1}{\partial y} + Q_1\,\frac{\partial f}{\partial y}\right) + \left(f\,\frac{\partial R_1}{\partial z} + R_1\,\frac{\partial f}{\partial z}\right)$$

$$= f\left(\frac{\partial P_1}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial R_1}{\partial z}\right) + \langle P_1, Q_1, R_1\rangle \cdot \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right\rangle = f\operatorname{div}\mathbf{F} + \mathbf{F}\cdot\nabla f$$

26. $\operatorname{curl}(f\mathbf{F}) = \left[\dfrac{\partial(fR_1)}{\partial y} - \dfrac{\partial(fQ_1)}{\partial z}\right]\mathbf{i} + \left[\dfrac{\partial(fP_1)}{\partial z} - \dfrac{\partial(fR_1)}{\partial x}\right]\mathbf{j} + \left[\dfrac{\partial(fQ_1)}{\partial x} - \dfrac{\partial(fP_1)}{\partial y}\right]\mathbf{k}$

$$= \left[f\,\frac{\partial R_1}{\partial y} + R_1\,\frac{\partial f}{\partial y} - f\,\frac{\partial Q_1}{\partial z} - Q_1\,\frac{\partial f}{\partial z}\right]\mathbf{i} + \left[f\,\frac{\partial P_1}{\partial z} + P_1\,\frac{\partial f}{\partial z} - f\,\frac{\partial R_1}{\partial x} - R_1\,\frac{\partial f}{\partial x}\right]\mathbf{j}$$

$$+ \left[f\,\frac{\partial Q_1}{\partial x} + Q_1\,\frac{\partial f}{\partial x} - f\,\frac{\partial P_1}{\partial y} - P_1\,\frac{\partial f}{\partial y}\right]\mathbf{k}$$

$$= f\left[\frac{\partial R_1}{\partial y} - \frac{\partial Q_1}{\partial z}\right]\mathbf{i} + f\left[\frac{\partial P_1}{\partial z} - \frac{\partial R_1}{\partial x}\right]\mathbf{j} + f\left[\frac{\partial Q_1}{\partial x} - \frac{\partial P_1}{\partial y}\right]\mathbf{k}$$

$$+ \left[R_1\frac{\partial f}{\partial y} - Q_1\frac{\partial f}{\partial z}\right]\mathbf{i} + \left[P_1\frac{\partial f}{\partial z} - R_1\frac{\partial f}{\partial x}\right]\mathbf{j} + \left[Q_1\frac{\partial f}{\partial x} - P_1\frac{\partial f}{\partial y}\right]\mathbf{k}$$

$$= f\operatorname{curl}\mathbf{F} + (\nabla f)\times\mathbf{F}$$

27. $\operatorname{div}(\mathbf{F}\times\mathbf{G}) = \nabla\cdot(\mathbf{F}\times\mathbf{G}) = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P_1 & Q_1 & R_1 \\ P_2 & Q_2 & R_2 \end{vmatrix} = \dfrac{\partial}{\partial x}\begin{vmatrix} Q_1 & R_1 \\ Q_2 & R_2 \end{vmatrix} - \dfrac{\partial}{\partial y}\begin{vmatrix} P_1 & R_1 \\ P_2 & R_2 \end{vmatrix} + \dfrac{\partial}{\partial z}\begin{vmatrix} P_1 & Q_1 \\ P_2 & Q_2 \end{vmatrix}$

$$= \left[Q_1\frac{\partial R_2}{\partial x} + R_2\frac{\partial Q_1}{\partial x} - Q_2\frac{\partial R_1}{\partial x} - R_1\frac{\partial Q_2}{\partial x}\right] - \left[P_1\frac{\partial R_2}{\partial y} + R_2\frac{\partial P_1}{\partial y} - P_2\frac{\partial R_1}{\partial y} - R_1\frac{\partial P_2}{\partial y}\right]$$

$$+ \left[P_1\frac{\partial Q_2}{\partial z} + Q_2\frac{\partial P_1}{\partial z} - P_2\frac{\partial Q_1}{\partial z} - Q_1\frac{\partial P_2}{\partial z}\right]$$

$$= \left[P_2\left(\frac{\partial R_1}{\partial y} - \frac{\partial Q_1}{\partial z}\right) + Q_2\left(\frac{\partial P_1}{\partial z} - \frac{\partial R_1}{\partial x}\right) + R_2\left(\frac{\partial Q_1}{\partial x} - \frac{\partial P_1}{\partial y}\right)\right]$$

$$- \left[P_1\left(\frac{\partial R_2}{\partial y} - \frac{\partial Q_2}{\partial z}\right) + Q_1\left(\frac{\partial P_2}{\partial z} - \frac{\partial R_2}{\partial x}\right) + R_1\left(\frac{\partial Q_2}{\partial x} - \frac{\partial P_2}{\partial y}\right)\right]$$

$$= \mathbf{G}\cdot\operatorname{curl}\mathbf{F} - \mathbf{F}\cdot\operatorname{curl}\mathbf{G}$$

28. $\operatorname{div}(\nabla f\times\nabla g) = \nabla g\cdot\operatorname{curl}(\nabla f) - \nabla f\cdot\operatorname{curl}(\nabla g)$ [by Exercise 27] $= \mathbf{0}$ [by Theorem 3]

29. $\operatorname{curl}(\operatorname{curl}\mathbf{F}) = \nabla \times (\nabla \times \mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial R_1/\partial y - \partial Q_1/\partial z & \partial P_1/\partial z - \partial R_1/\partial x & \partial Q_1/\partial x - \partial P_1/\partial y \end{vmatrix}$

$$= \left(\frac{\partial^2 Q_1}{\partial y \partial x} - \frac{\partial^2 P_1}{\partial y^2} - \frac{\partial^2 P_1}{\partial z^2} + \frac{\partial^2 R_1}{\partial z \partial x}\right)\mathbf{i} + \left(\frac{\partial^2 R_1}{\partial z \partial y} - \frac{\partial^2 Q_1}{\partial z^2} - \frac{\partial^2 Q_1}{\partial x^2} + \frac{\partial^2 P_1}{\partial x \partial y}\right)\mathbf{j}$$
$$+ \left(\frac{\partial^2 P_1}{\partial x \partial z} - \frac{\partial^2 R_1}{\partial x^2} - \frac{\partial^2 R_1}{\partial y^2} + \frac{\partial^2 Q_1}{\partial y \partial z}\right)\mathbf{k}$$

Now let's consider $\operatorname{grad}(\operatorname{div}\mathbf{F}) - \nabla^2\mathbf{F}$ and compare with the above.

(Note that $\nabla^2\mathbf{F}$ is defined on page 1102 [ET 1066].)

$$\operatorname{grad}(\operatorname{div}\mathbf{F}) - \nabla^2\mathbf{F} = \left[\left(\frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 Q_1}{\partial x \partial y} + \frac{\partial^2 R_1}{\partial x \partial z}\right)\mathbf{i} + \left(\frac{\partial^2 P_1}{\partial y \partial x} + \frac{\partial^2 Q_1}{\partial y^2} + \frac{\partial^2 R_1}{\partial y \partial z}\right)\mathbf{j} + \left(\frac{\partial^2 P_1}{\partial z \partial x} + \frac{\partial^2 Q_1}{\partial z \partial y} + \frac{\partial^2 R_1}{\partial z^2}\right)\mathbf{k}\right]$$
$$- \left[\left(\frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_1}{\partial y^2} + \frac{\partial^2 P_1}{\partial z^2}\right)\mathbf{i} + \left(\frac{\partial^2 Q_1}{\partial x^2} + \frac{\partial^2 Q_1}{\partial y^2} + \frac{\partial^2 Q_1}{\partial z^2}\right)\mathbf{j}\right.$$
$$\left. + \left(\frac{\partial^2 R_1}{\partial x^2} + \frac{\partial^2 R_1}{\partial y^2} + \frac{\partial^2 R_1}{\partial z^2}\right)\mathbf{k}\right]$$
$$= \left(\frac{\partial^2 Q_1}{\partial x \partial y} + \frac{\partial^2 R_1}{\partial x \partial z} - \frac{\partial^2 P_1}{\partial y^2} - \frac{\partial^2 P_1}{\partial z^2}\right)\mathbf{i} + \left(\frac{\partial^2 P_1}{\partial y \partial x} + \frac{\partial^2 R_1}{\partial y \partial z} - \frac{\partial^2 Q_1}{\partial x^2} - \frac{\partial^2 Q_1}{\partial z^2}\right)\mathbf{j}$$
$$+ \left(\frac{\partial^2 P_1}{\partial z \partial x} + \frac{\partial^2 Q_1}{\partial z \partial y} - \frac{\partial^2 R_1}{\partial x^2} - \frac{\partial^2 R_2}{\partial y^2}\right)\mathbf{k}$$

Then applying Clairaut's Theorem to reverse the order of differentiation in the second partial derivatives as needed and comparing, we have $\operatorname{curl}\operatorname{curl}\mathbf{F} = \operatorname{grad}\operatorname{div}\mathbf{F} - \nabla^2\mathbf{F}$ as desired.

32. $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k} \quad \Rightarrow \quad r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$, so

$$\mathbf{F} = \frac{\mathbf{r}}{r^p} = \frac{x}{(x^2+y^2+z^2)^{p/2}}\,\mathbf{i} + \frac{y}{(x^2+y^2+z^2)^{p/2}}\,\mathbf{j} + \frac{z}{(x^2+y^2+z^2)^{p/2}}\,\mathbf{k}$$

Then $\dfrac{\partial}{\partial x}\dfrac{x}{(x^2+y^2+z^2)^{p/2}} = \dfrac{(x^2+y^2+z^2) - px^2}{(x^2+y^2+z^2)^{1+p/2}} = \dfrac{r^2 - px^2}{r^{p+2}}$. Similarly,

$\dfrac{\partial}{\partial y}\dfrac{y}{(x^2+y^2+z^2)^{p/2}} = \dfrac{r^2 - py^2}{r^{p+2}}$ and $\dfrac{\partial}{\partial z}\dfrac{z}{(x^2+y^2+z^2)^{p/2}} = \dfrac{r^2 - pz^2}{r^{p+2}}$. Thus

$$\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{r^2 - px^2}{r^{p+2}} + \frac{r^2 - py^2}{r^{p+2}} + \frac{r^2 - pz^2}{r^{p+2}} = \frac{3r^2 - px^2 - py^2 - pz^2}{r^{p+2}}$$
$$= \frac{3r^2 - p(x^2+y^2+z^2)}{r^{p+2}} = \frac{3r^2 - pr^2}{r^{p+2}} = \frac{3-p}{r^p}$$

Consequently, if $p = 3$ we have $\operatorname{div}\mathbf{F} = 0$.

33. By (13), $\oint_C f(\nabla g)\cdot\mathbf{n}\,ds = \iint_D \operatorname{div}(f\nabla g)\,dA = \iint_D [f\operatorname{div}(\nabla g) + \nabla g\cdot\nabla f]\,dA$ by Exercise 25. But $\operatorname{div}(\nabla g) = \nabla^2 g$. Hence $\iint_D f\nabla^2 g\,dA = \oint_C f(\nabla g)\cdot\mathbf{n}\,ds - \iint_D \nabla g\cdot\nabla f\,dA$.

34. By Exercise 33, $\iint_D f\nabla^2 g\,dA = \oint_C f(\nabla g)\cdot\mathbf{n}\,ds - \iint_D \nabla g\cdot\nabla f\,dA$ and
$\iint_D g\nabla^2 f\,dA = \oint_C g(\nabla f)\cdot\mathbf{n}\,ds - \iint_D \nabla f\cdot\nabla g\,dA$. Hence
$\iint_D \left(f\nabla^2 g - g\nabla^2 f\right)dA = \oint_C [f(\nabla g)\cdot\mathbf{n} - g(\nabla f)\cdot\mathbf{n}]\,ds + \iint_D (\nabla f\cdot\nabla g - \nabla g\cdot\nabla f)\,dA = \oint_C [f\nabla g - g\nabla f]\cdot\mathbf{n}\,ds$.

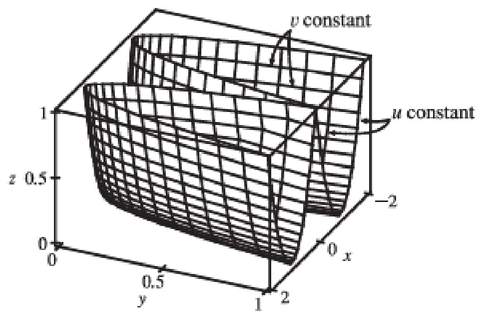
39. For any continuous function f on $\mathbb{R}^3$, define a vector field $\mathbf{G}(x,y,z) = \langle g(x,y,z), 0, 0\rangle$ where $g(x,y,z) = \int_0^x f(t,y,z)\,dt$. Then $\operatorname{div}\mathbf{G} = \frac{\partial}{\partial x}(g(x,y,z)) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(0) = \frac{\partial}{\partial x}\int_0^x f(t,y,z)\,dt = f(x,y,z)$ by the Fundamental Theorem of Calculus. Thus every continuous function f on $\mathbb{R}^3$ is the divergence of some vector field.

3. $\mathbf{r}(u,v) = (u+v)\,\mathbf{i} + (3-v)\,\mathbf{j} + (1+4u+5v)\,\mathbf{k} = \langle 0,3,1\rangle + u\,\langle 1,0,4\rangle + v\,\langle 1,-1,5\rangle$. From Example 3, we recognize this as a vector equation of a plane through the point $(0,3,1)$ and containing vectors $\mathbf{a} = \langle 1,0,4\rangle$ and $\mathbf{b} = \langle 1,-1,5\rangle$. If we wish to find a more conventional equation for the plane, a normal vector to the plane is $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 4 \\ 1 & -1 & 5 \end{vmatrix} = 4\,\mathbf{i} - \mathbf{j} - \mathbf{k}$ and an equation of the plane is $4(x-0) - (y-3) - (z-1) = 0$ or $4x - y - z = -4$.

6. $\mathbf{r}(s,t) = s\sin 2t\,\mathbf{i} + s^2\,\mathbf{j} + s\cos 2t\,\mathbf{k}$, so the corresponding parametric equations for the surface are $x = s\sin 2t$, $y = s^2$, $z = s\cos 2t$. For any point (x,y,z) on the surface, we have $x^2 + z^2 = s^2\sin^2 2t + s^2\cos^2 2t = s^2 = y$. Since no restrictions are placed on the parameters, the surface is $y = x^2 + z^2$, which we recognize as a circular paraboloid whose axis is the y-axis.

8. $\mathbf{r}(u,v) = \langle u+v, u^2, v^2\rangle$, $-1\le u\le 1$, $-1\le v\le 1$.

The surface has parametric equations $x = u+v$, $y = u^2$, $z = v^2$, $-1\le u\le 1$, $-1\le v\le 1$. If $u = u_0$ is constant, $y = u_0^2 =$ constant, so the corresponding grid curves are the curves parallel to the xz-plane. If $v = v_0$ is constant, $z = v_0^2 =$ constant, so the corresponding grid curves are the curves parallel to the xy-plane.



13. $\mathbf{r}(u,v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + v\,\mathbf{k}$. The parametric equations for the surface are $x = u\cos v$, $y = u\sin v$, $z = v$. We look at the grid curves first; if we fix v, then x and y parametrize a straight line in the plane $z = v$ which intersects the z-axis. If u is held constant, the projection onto the xy-plane is circular; with $z = v$, each grid curve is a helix. The surface is a spiraling ramp, graph I.

14. $\mathbf{r}(u,v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + \sin u\,\mathbf{k}$. The corresponding parametric equations for the surface are $x = u\cos v$, $y = u\sin v$, $z = \sin u$, $-\pi \le u \le \pi$. If $u = u_0$ is held constant, then $x = u_0\cos v$, $y = u_0\sin v$ so each grid curve is a circle of radius $|u_0|$ in the horizontal plane $z = \sin u_0$. If $v = v_0$ is constant, then $x = u\cos v_0$, $y = u\sin v_0 \quad\Rightarrow\quad y = (\tan v_0)x$, so the grid curves lie in vertical planes $y = kx$ through the z-axis. In fact, since x and y are constant multiples of u and $z = \sin u$, each of these traces is a sine wave. The surface is graph I.

15. $\mathbf{r}(u,v) = \sin v\,\mathbf{i} + \cos u\,\sin 2v\,\mathbf{j} + \sin u\,\sin 2v\,\mathbf{k}$. Parametric equations for the surface are $x = \sin v$, $y = \cos u\,\sin 2v$, $z = \sin u\,\sin 2v$. If $v = v_0$ is fixed, then $x = \sin v_0$ is constant, and $y = (\sin 2v_0)\cos u$ and $z = (\sin 2v_0)\sin u$ describe a circle of radius $|\sin 2v_0|$, so each corresponding grid curve is a circle contained in the vertical plane $x = \sin v_0$ parallel to the yz-plane. The only possible surface is graph II. The grid curves we see running lengthwise along the surface correspond to holding u constant, in which case $y = (\cos u_0)\sin 2v$, $z = (\sin u_0)\sin 2v \quad\Rightarrow\quad z = (\tan u_0)y$, so each grid curve lies in a plane $z = ky$ that includes the x-axis.

16. $x = (1-u)(3+\cos v)\cos 4\pi u$, $y = (1-u)(3+\cos v)\sin 4\pi u$, $z = 3u + (1-u)\sin v$. These equations correspond to graph VI: when $u = 0$, then $x = 3 + \cos v$, $y = 0$, and $z = \sin v$, which are equations of a circle with radius 1 in the xz-plane centered at $(3,0,0)$. When $u = \frac{1}{2}$, then $x = \frac{3}{2} + \frac{1}{2}\cos v$, $y = 0$, and $z = \frac{3}{2} + \frac{1}{2}\sin v$, which are equations of a circle with radius $\frac{1}{2}$ in the xz-plane centered at $\left(\frac{3}{2}, 0, \frac{3}{2}\right)$. When $u = 1$, then $x = y = 0$ and $z = 3$, giving the topmost point shown in the graph. This suggests that the grid curves with u constant are the vertically oriented circles visible on the surface. The spiralling grid curves correspond to keeping v constant.

17. $x = \cos^3 u\,\cos^3 v$, $y = \sin^3 u\,\cos^3 v$, $z = \sin^3 v$. If $v = v_0$ is held constant then $z = \sin^3 v_0$ is constant, so the corresponding grid curve lies in a horizontal plane. Several of the graphs exhibit horizontal grid curves, but the curves for this surface are neither circles nor straight lines, so graph III is the only possibility. (In fact, the horizontal grid curves here are members of the family $x = a\cos^3 u$, $y = a\sin^3 u$ and are called astroids.) The vertical grid curves we see on the surface correspond to $u = u_0$ held constant, as then we have $x = \cos^3 u_0\,\cos^3 v$, $y = \sin^3 u_0\,\cos^3 v$ so the corresponding grid curve lies in the vertical plane $y = (\tan^3 u_0)x$ through the z-axis.

18. $x = (1-|u|)\cos v$, $y = (1-|u|)\sin v$, $z = u$. Then $x^2 + y^2 = (1-|u|)^2\cos^2 v + (1-|u|)^2\sin^2 v = (1-|u|)^2$, so if u is held constant, each grid curve is a circle of radius $(1-|u|)$ in the horizontal plane $z = u$. The graph then must be graph VI. If v is held constant, so $v = v_0$, we have $x = (1-|u|)\cos v_0$ and $y = (1-|u|)\sin v_0$. Then $y = (\tan v_0)x$, so the grid curves we see running vertically along the surface in the planes $y = kx$ correspond to keeping v constant.

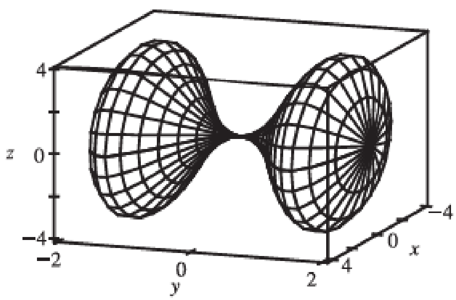
19. From Example 3, parametric equations for the plane through the point $(1,2,-3)$ that contains the vectors $\mathbf{a} = \langle 1,1,-1\rangle$ and $\mathbf{b} = \langle 1,-1,1\rangle$ are $x = 1 + u(1) + v(1) = 1 + u + v$, $y = 2 + u(1) + v(-1) = 2 + u - v$, $z = -3 + u(-1) + v(1) = -3 - u + v$.

22. $x = 4 - y^2 - 2z^2$, $y = y$, $z = z$ where $y^2 + 2z^2 \le 4$ since $x \ge 0$. Then the associated vector equation is $\mathbf{r}(y,z) = (4 - y^2 - 2z^2)\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.

24. In spherical coordinates, parametric equations are $x = 4\sin\phi\cos\theta$, $y = 4\sin\phi\sin\theta$, $z = 4\cos\phi$. The intersection of the sphere with the plane $z = 2$ corresponds to $z = 4\cos\phi = 2 \quad\Rightarrow\quad \cos\phi = \frac{1}{2} \quad\Rightarrow\quad \phi = \frac{\pi}{3}$. By symmetry, the intersection of the sphere with the plane $z = -2$ corresponds to $\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Thus the surface is described by $0 \le \theta \le 2\pi$, $\frac{\pi}{3} \le \phi \le \frac{2\pi}{3}$.

26. Using x and y as the parameters, $x = x$, $y = y$, $z = x + 3$ where $0 \le x^2 + y^2 \le 1$. Also, since the plane intersects the cylinder in an ellipse, the surface is a planar ellipse in the plane $z = x + 3$. Thus, parametrizing with respect to s and θ, we have $x = s\cos\theta$, $y = s\sin\theta$, $z = 3 + s\cos\theta$ where $0 \le s \le 1$ and $0 \le \theta \le 2\pi$.

30. Letting θ be the angle of rotation about the y-axis, we have the parametrization $x = (4y^2 - y^4)\cos\theta$, $\; y = y$, $z = (4y^2 - y^4)\sin\theta$, $\; -2 \le y \le 2$, $\; 0 \le \theta \le 2\pi$.



34. $\mathbf{r}(u,v) = u^2\,\mathbf{i} + v^2\,\mathbf{j} + uv\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}(1,1) = (1,1,1)$.
$\mathbf{r}_u = 2u\,\mathbf{i} + v\,\mathbf{k}$ and $\mathbf{r}_v = 2v\,\mathbf{j} + u\,\mathbf{k}$, so a normal vector to the surface at the point $(1,1,1)$ is
$\mathbf{r}_u(1,1) \times \mathbf{r}_v(1,1) = (2\,\mathbf{i} + \mathbf{k}) \times (2\,\mathbf{j} + \mathbf{k}) = -2\,\mathbf{i} - 2\,\mathbf{j} + 4\,\mathbf{k}$.
Thus an equation of the tangent plane at the point $(1,1,1)$ is
$-2(x-1) - 2(y-1) + 4(z-1) = 0$ or $x + y - 2z = 0$.