

Question 1 (30 sec / 30 sec)

Math 317 - End of Course Review Questions

Fall 2010

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A planimeter is...

- A. a unit of flux
- B. a measuring device for area based on Green's Theorem
- C. a measuring device for flux based on the Divergence Theorem
- D. a unit of area
- E. 10^{12} metres



Question 3 (1 min / 1 min)

Let $f(x, y, z)$ be a scalar function and \mathbf{F} a 3D vector field. Use the '3D nabla' $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$.

Which of the following is nonsense notation?

- A. $\nabla \cdot (\nabla \times \mathbf{F})$
- B. $\nabla \times (\nabla f)$
- C. $\nabla \cdot (\nabla \cdot \mathbf{F})$
- D. $\nabla(\nabla \cdot \mathbf{F})$
- E. $\nabla \cdot (\nabla f)$



Question 6 (1 min / 1 min)

A hamster cage is a 1 m^3 cube. The bottom of the cage is solid and no air can pass through. The sides and top of the cage are mesh. There's a draft in the room, and you measure the air flux through each of the side panels of the cage to be 1 g/hr inward toward the hamsters. If the air pressure is changing, so that the divergence everywhere in the room is $1\text{ g}/(\text{m}^3\text{hr})$, then what is the flux out through the top panel of the cage?

- A. 2 gr/hr
- B. 3 gr/hr
- C. 4 gr/hr
- D. 5 gr/hr
- E. 6 gr/hr



Question 2 (2 min / 2 min)

You are an evil hamster farmer. Your hamsters are forced to run in hamster wheels to generate electricity for your hamsterburger factory. A hamster wheel S is given by

$$x^2 + z^2 = \sqrt{y}, \quad 4 < y < 9$$

The hamster wheel is oriented inward, i.e. the normal vectors point in toward the running hamster (to better poke their poor little feet). Which of the following is ∂S , oriented so that Stokes' Theorem holds for S and ∂S ?

Let $C_1 : \mathbf{r}(t) = (\sqrt{2} \cos t, 4, \sqrt{2} \sin t), \quad 0 \leq t \leq 2\pi$.
Let $C_2 : \mathbf{r}(t) = (\sqrt{3} \cos t, 9, \sqrt{3} \sin t), \quad 0 \leq t \leq 2\pi$.

- A. $C_1 \cup C_2$
- B. $C_1 \cup -C_2$
- C. $-C_1 \cup C_2$
- D. $-C_1 \cup -C_2$
- E. Yummmmm, hamsters.



Question 5 (1 min / 1 min)

The hamsters are getting smarter. They don't want to do any work while they run around their hamster wheels, so they decide they need a conservative vector field (they're not exactly sure what they plan to do with it when they get it though). Which of the following vector fields is **not** conservative?

- A. $\mathbf{F} = \langle x, y \rangle$
- B. $\mathbf{F} = \langle \cos(x^2), \cos(y^2) \rangle$
- C. $\mathbf{F} = \langle y, x \rangle$
- D. $\mathbf{F} = \langle xy, xy \rangle$
- E. $\mathbf{F} = \langle 2, 2 \rangle$



Question 8 (4 min / 3 min)

Four of the following functions parametrise the same curve. One parametrises a different curve. Which is the odd one out?

- A. $\mathbf{r}(t) = \langle t \cos t, t \sin t, t \rangle, \quad 0 \leq t \leq 1$
- B. $\mathbf{r}(t) = \langle t \sin t, t \cos t, t \rangle, \quad 0 \leq t \leq 1$
- C. $\mathbf{r}(t) = \langle (\pi - t) \cos t, (\pi - t) \sin t, t - \pi \rangle, \quad \pi \leq t \leq \pi + 1$
- D. $\mathbf{r}(t) = \langle t \sin(t + \frac{\pi}{2}), t \sin t, t \rangle, \quad 0 \leq t \leq 1$
- E. $\mathbf{r}(t) = \langle t^3 \cos(t^3), t^3 \sin(t^3), t^3 \rangle, \quad 0 \leq t \leq 1$



Question 4 (1 min / 1 min)

The Stuffed Hamster Division of your Culinary Research Institute has been stuck on this problem for a while. In which of the following vector fields will the flux through all closed (possibly hamster-shaped) surfaces be zero?

- A. $\mathbf{F} = \langle x, y, z \rangle$
- B. $\mathbf{F} = \langle y, z, x \rangle$
- C. $\mathbf{F} = \langle z, x, y \rangle$
- D. A and C
- E. B and C



Question 7 (1 min / 1 min)

Which of the following quantities does not depend on the parametrisation of the surface, but only on the surface itself. (Assume we only allow smooth parametrisations.)

- A. the tangent plane
- B. the orientation
- C. the normal $\mathbf{r}_u \times \mathbf{r}_v$
- D. the grid curves
- E. how happy you are

Note: Answer E is not the correct answer.



Question 9 (4 min / 3 min)

Four of the following functions parametrise the same surface. One parametrises a different surface. Which is the odd one out?

- A. $\mathbf{s}(u, v) = (u \cos(v), u \sin(v), u^2)$, $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$
- B. $\mathbf{s}(u, v) = (u \sin(v), -u \cos(v), u^2)$, $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$
- C. $\mathbf{s}(u, v) = (\sqrt{u} \cos(v), \sqrt{u} \sin(v), u)$, $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$
- D. $\mathbf{s}(u, v) = (u \cos(v^2), u \sin(v^2), u^2)$, $0 \leq u \leq 1$, $0 \leq v \leq \sqrt{2\pi}$
- E. $\mathbf{s}(u, v) = (u, v, u^2 + v^2)$, $0 \leq u \leq 1$, $0 \leq v \leq 1$



Question 10 (2 min / 1 min)

Which field has zero 2D flux ($\int_C \mathbf{F} \cdot \mathbf{n} ds$) through the unit circle?

- A. $\mathbf{F} = \langle x, y \rangle$
- B. $\mathbf{F} = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$
- C. $\mathbf{F} = \langle x^2, y^2 \rangle$
- D. $\mathbf{F} = \langle -x, -y \rangle$
- E. $\mathbf{F} = \langle x^2 - 2xy, y^2 - 2xy \rangle$



Question 11 (2 min / 2 min)

You are designing a new hamsterburger patty shape, meant to hold lots of ketchup. As part of your calculations, you need to find the curve of intersection of the two surfaces

$$x^2 + y^2 = 4, \quad z = x^2$$

The intersection could be parametrised by...

- A. $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, 4 - 4 \sin^2 t \rangle$, $0 < t < 2\pi$
- B. $\mathbf{r}(t) = \langle 2 \cos t, 2 \sin t, \cos^2 t \rangle$, $0 < t < 2\pi$
- C. $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, $0 < t < 2\pi$
- D. $\mathbf{r}(t) = \langle 4 \cos t, 4 \sin t, 4 \cos^2 t \rangle$, $0 < t < 2\pi$
- E. $\mathbf{r}(t) = \langle 4 \cos t, 4 \sin t, 0 \rangle$, $0 < t < 2\pi$



Question 12 (3 min / 1 min)

Your hamster factory suffers a revolt! You are the origin of the universe (of course) and the hamsters lob a hamster bomb at you from coordinates $(1, 0, 0)$. The surface of your planet is the xy plane and acceleration due to gravity is exactly $(0, 0, -1)$ (what a convenient planet). Fortunately, hamsters are gullible. They ask you first which initial velocity they should use in order to hit you. Out of self preservation, you lie and say

- A. $\mathbf{v}_0 = \langle -\frac{1}{6}, 0, 3 \rangle$
- B. $\mathbf{v}_0 = \langle -1, 0, \frac{1}{2} \rangle$
- C. $\mathbf{v}_0 = \langle -1, 0, 1 \rangle$
- D. $\mathbf{v}_0 = \langle -2, 0, \frac{1}{4} \rangle$
- E. $\mathbf{v}_0 = \langle -\frac{1}{2}, 0, 1 \rangle$



Question 13 (2 min / 1 min)

The hamsters dream one day of opening a hair salon when they escape from your hamster compound. They want to specialise in frizzy perms. They ask you, which of the following vector fields is **not** the curl of another vector field?

- A. $\mathbf{F} = \langle z, z, y \rangle$
- B. $\mathbf{F} = \langle x^2, -2xy, 5 \rangle$
- C. $\mathbf{F} = \langle 3x, -2y, -z \rangle$
- D. $\mathbf{F} = \langle x, y, z \rangle$
- E. $\mathbf{F} = \langle 2, 2, 4 \rangle$

