Math 317 - End of Course Review Questions

Fall 2010

December 3, 2010

Question 1 (30 sec / 30 sec)

A planimeter is...

A. a unit of flux

B. a measuring device for area based on Green's Theorem

C. a measuring device for flux based on the Divergence Theorem

D. a unit of area

E. 10^{12} metres

Question 2 (2 min / 2 min)

You are an evil hamster farmer. Your hamsters are forced to run in hamster wheels to generate electricity for your hamsterburger factory. A hamster wheel S is given by

$$x^2 + z^2 = \sqrt{y}, \quad 4 < y < 9$$

The hamster wheel is oriented inward, i.e. the normal vectors point in toward the running hamster (to better poke their poor little feet). Which of the following is ∂S, oriented so that Stokes' Theorem holds for S and ∂S?

Let $C_1 : \mathbf{r}(t) = \langle \sqrt{2}\cos t, 4, \sqrt{2}\sin t \rangle, \quad 0 \le t \le 2\pi$.
Let $C_2 : \mathbf{r}(t) = \langle \sqrt{3}\cos t, 9, \sqrt{3}\sin t \rangle, \quad 0 \le t \le 2\pi$.

A. $C_1 \cup C_2$

B. $C_1 \cup -C_2$

C. $-C_1 \cup C_2$

D. $-C_1 \cup -C_2$

E. Yummmmm, hamsters.

Question 3 (1 min / 1 min)

Let $f(x, y, z)$ be a scalar function and $\mathbf{F}$ a 3D vector field. Use the '3D nabla' $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$.
Which of the following is nonsense notation?

A. $\nabla \cdot (\nabla \times \mathbf{F})$

B. $\nabla \times (\nabla f)$

C. $\nabla \cdot (\nabla \cdot \mathbf{F})$

D. $\nabla(\nabla \cdot \mathbf{F})$

E. $\nabla \cdot (\nabla f)$

Question 4 (1 min / 1 min)

The Stuffed Hamster Division of your Culinary Research Institute has been stuck on this problem for a while. In which of the following vector fields will the flux through all closed (possibly hamster-shaped) surfaces be zero?

A. $\mathbf{F} = \langle x, y, z \rangle$
B. $\mathbf{F} = \langle y, z, x \rangle$
C. $\mathbf{F} = \langle z, x, y \rangle$
D. A and C
E. B and C

Question 5 (1 min / 1 min)

The hamsters are getting smarter. They don't want to do any work while they run around their hamster wheels, so they decide they need a conservative vector field (they're not exactly sure what they plan to do with it when they get it though). Which of the following vector fields is **not** conservative?

A. $\mathbf{F} = \langle x, y \rangle$
B. $\mathbf{F} = \langle \cos(x^2), \cos(y^2) \rangle$
C. $\mathbf{F} = \langle y, x \rangle$
D. $\mathbf{F} = \langle xy, xy \rangle$
E. $\mathbf{F} = \langle 2, 2 \rangle$

Question 6 (1 min / 1 min)

A hamster cage is a $1m^3$ cube. The bottom of the cage is solid and no air can pass through. The sides and top of the cage are mesh. There's a draft in the room, and you measure the air flux through each of the side panels of the cage to be $1g/hr$ inward toward the hamsters. If the air pressure is changing, so that the divergence everywhere in the room is $1g/(m^3hr)$, then what is the flux out through the top panel of the cage?

A. 2 gr/hr
B. 3 gr/hr
C. 4 gr/hr
D. 5 gr/hr
E. 6 gr/hr

Question 7 (1 min / 1 min)

Which of the following quantities does not depend on the parametrisation of the surface, but only on the surface itself. (Assume we only allow smooth parametrisations.)

A. the tangent plane

B. the orientation

C. the normal $\mathbf{r}_u \times \mathbf{r}_v$

D. the grid curves

E. how happy you are

Note: Answer E is not the correct answer.

Question 8 (4 min / 3 min)

Four of the following functions parametrise the same curve. One parametrises a different curve. Which is the odd one out?

A. $\mathbf{r}(t) = \langle t\cos t, t\sin t, t\rangle, \quad 0 \le t \le 1$

B. $\mathbf{r}(t) = \langle t\sin t, t\cos t, t\rangle, \quad 0 \le t \le 1$

C. $\mathbf{r}(t) = \langle (\pi - t)\cos t, (\pi - t)\sin t, t - \pi\rangle, \quad \pi \le t \le \pi + 1$

D. $\mathbf{r}(t) = \langle t\sin(t + \frac{\pi}{2}), t\sin t, t\rangle, \quad 0 \le t \le 1$

E. $\mathbf{r}(t) = \langle t^3\cos(t^3), t^3\sin(t^3), t^3\rangle, \quad 0 \le t \le 1$

Question 9 (4 min / 3 min)

Four of the following functions parametrise the same surface. One parametrises a different surface. Which is the odd one out?

A. $\mathbf{s}(u, v) = \langle u\cos(v), u\sin(v), u^2\rangle,$
$0 \le u \le 1, \quad 0 \le v \le 2\pi$

B. $\mathbf{s}(u, v) = \langle u\sin(v), -u\cos(v), u^2\rangle,$
$0 \le u \le 1, \quad 0 \le v \le 2\pi$

C. $\mathbf{s}(u, v) = \langle \sqrt{u}\cos(v), \sqrt{u}\sin(v), u\rangle,$
$0 \le u \le 1, \quad 0 \le v \le 2\pi$

D. $\mathbf{s}(u, v) = \langle u\cos(v^2), u\sin(v^2), u^2\rangle,$
$0 \le u \le 1, \quad 0 \le v \le \sqrt{2\pi}$

E. $\mathbf{s}(u, v) = \langle u, v, u^2 + v^2\rangle,$
$0 \le u \le 1, \quad 0 \le v \le 1$

Question 10 (2 min / 1 min)

Which field has zero 2D flux ($\int_C \mathbf{F} \cdot \mathbf{n} ds$) through the unit circle?

A. $\mathbf{F} = \langle x, y \rangle$

B. $\mathbf{F} = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$

C. $\mathbf{F} = \langle x^2, y^2 \rangle$

D. $\mathbf{F} = \langle -x, -y \rangle$

E. $\mathbf{F} = \langle x^2 - 2xy, y^2 - 2xy \rangle$

Question 11 (2 min / 2 min)

You are designing a new hamsterburger patty shape, meant to hold lots of ketchup. As part of your calculations, you need to find the curve of intersection of the two surfaces

$$x^2 + y^2 = 4, \quad z = x^2$$

The intersection could be parametrised by...

A. $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 4 - 4\sin^2 t\rangle, \quad 0 < t < 2\pi$
B. $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, \cos^2 t\rangle, \quad 0 < t < 2\pi$
C. $\mathbf{r}(t) = \langle \cos t, \sin t, t\rangle, \quad 0 < t < 2\pi$
D. $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 4\cos^2 t\rangle, \quad 0 < t < 2\pi$
E. $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 0\rangle, \quad 0 < t < 2\pi$

Question 12 (3 min / 1 min)

Your hamster factory suffers a revolt! You are the origin of the universe (of course) and the hamsters lob a hamster bomb at you from coordinates $(1, 0, 0)$. The surface of your planet is the *xy* plane and acceleration due to gravity is exactly $\langle 0, 0, -1\rangle$ (what a convenient planet). Fortunately, hamsters are gullible. They ask you first which initial velocity they should use in order to hit you. Out of self preservation, you lie and say

A. $\mathbf{v}_0 = \langle -\frac{1}{6}, 0, 3\rangle$

B. $\mathbf{v}_0 = \langle -1, 0, \frac{1}{2}\rangle$

C. $\mathbf{v}_0 = \langle -1, 0, 1\rangle$

D. $\mathbf{v}_0 = \langle -2, 0, \frac{1}{4}\rangle$

E. $\mathbf{v}_0 = \langle -\frac{1}{2}, 0, 1\rangle$

Question 13 (2 min / 1 min)

The hamsters dream one day of opening a hair salon when they escape from your hamster compound. They want to specialise in frizzy perms. They ask you, which of the following vector fields is **not** the curl of another vector field?

A. $\mathbf{F} = \langle z, z, y \rangle$
B. $\mathbf{F} = \langle x^2, -2xy, 5 \rangle$
C. $\mathbf{F} = \langle 3x, -2y, -z \rangle$
D. $\mathbf{F} = \langle x, y, z \rangle$
E. $\mathbf{F} = \langle 2, 2, 4 \rangle$