2. Show that if κ and λ are infinite cardinals and $\kappa < \lambda$, then there is an infinite cardinal μ such that $\mu^\kappa < \mu^\lambda$.

Proof. Let μ be the limit of the sequence $(\mu_\alpha)_{\alpha<\kappa^+}$, which is defined below by transfinite recursion:

- $\mu_0 = \lambda$,
- $\mu_1 = 2^\lambda$,
- $\mu_{\alpha+1} = (\mu_\alpha^+)^\lambda$,
- for any limit ordinal α, $\mu_\alpha = \bigcup_{\beta<\alpha} \mu_\beta$.

Clearly μ is a strong limit cardinal since whenever $\eta < \mu$, $\eta < \mu_\alpha$ for some $\alpha \in \mathrm{On}$, hence $2^\eta \leq (\mu_\alpha)^\eta \leq (\mu_\alpha^+)^\eta = \mu_{\alpha+1} < \mu$. Moreover, $\mathrm{cf}(\mu) = \kappa^+$ since μ is the limit of a strictly increasing κ^+- sequence of cardinals. It follows that $\mathrm{cf}(\mu) > \kappa$, and $\kappa^+ \leq \lambda$ since there are no cardinals in (κ, κ^+). Finally, μ is λ-unreachable from below since for any $\eta \leq \mu_\alpha < \mu$, with $\alpha \in \mathrm{On}$, $\eta^\lambda \leq (\mu_\alpha)^\lambda \leq (\mu_\alpha^+)^\lambda = \mu_{\alpha+1} < \mu$. By part *(iii)* of the Main Theorem of Cardinal Arithmetic, $\mu^\lambda = \mu^{\mathrm{cf}(\mu)}$ and $\mu = \mu^\kappa$, and by Kőnig's Theorem, $\mu^\lambda = \mu^{\mathrm{cf}(\mu)} > \mu = \mu^\kappa$. □