

HW 3.

1. (Bender-Stone, Green, Rodriguez) Show that the following statement is equivalent over ZFC to CH:

The real plane \mathbb{R}^2 may be partitioned into two cells, $\{A, B\}$, in such a way that any horizontal line in \mathbb{R}^2 has countable intersection with A and any vertical line in \mathbb{R}^2 has countable intersection with B .

2. (Bender-Stone) Show that if κ and λ are infinite cardinals and $\kappa < \lambda$, then there is an infinite cardinal μ such that $\mu^\kappa < \mu^\lambda$.
3. (Green) Show that the following statement is equivalent over ZFC to GCH:
If κ is any infinite cardinal, then $\kappa^{\text{cf}(\kappa)} = \kappa^+$.
4. (Rodriguez) Let \mathbb{F} be a field of size κ and let V be an \mathbb{F} -vector space of infinite dimension λ .
 - (a) Show that the dimension of the dual space V^* is κ^λ .
 - (b) Use the Main Theorem of Cardinal Arithmetic to simplify $\dim_{\mathbb{F}}(V^*)$ in the case where $|\mathbb{F}| = \beth_{\omega_1}$ and $\dim_{\mathbb{F}}(V) = \aleph_0$.