

HW 2.

1. (Bender-Stone, Green) Find all pairs (κ, λ) of cardinals such that the cardinal sum $\kappa +_c \lambda$ agrees with the ordinal sum $\kappa +_o \lambda$.
2. (Bender-Stone, Rodriguez) Show that
 - (a) $|\mathcal{P}(\aleph_0)| = 2^{\aleph_0}$.
 - (b) Show that the ordered set $\langle \mathcal{P}(\aleph_0); \subseteq \rangle$ contains a chain of cardinality 2^{\aleph_0} .
 - (c) Show that the ordered set $\langle \mathcal{P}(\aleph_0); \subseteq \rangle$ contains an antichain of cardinality 2^{\aleph_0} .
3. (Green, Rodriguez)
 - (a) Suppose that κ is an infinite cardinal and α is a smaller ordinal. Show that the interval $[\alpha, \kappa)$ of ordinals is order-isomorphic to κ .
 - (b) Suppose that $\kappa_0 < \kappa_1 < \kappa_2 < \cdots$ is a strictly increasing sequence of cardinals with limit κ . Explain why each interval $[\kappa_i, \kappa_{i+1})$ is order-isomorphic to κ_{i+1} .
 - (c) Suppose that $\kappa_0 < \kappa_1 < \kappa_2 < \cdots$ is an increasing sequence of cardinals with limit κ . Show that $\sum \kappa_i = \sup\{\kappa_i\} = \kappa$.