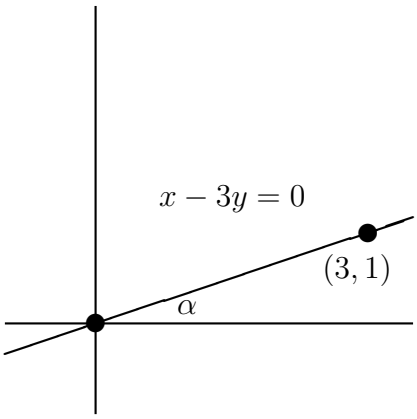
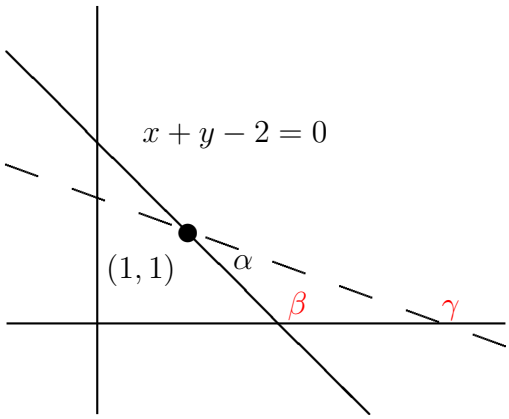
Practice.

This exercise helps to understand how Cartesian planes satisfy congruence axioms for angles.





Task: Find the equation of the dashed line, which is meant to form an angle of α with the solid line.

Since the point $(1, 1)$ lies on the dashed line, the point-slope equation for the line is $y - 1 = m(x - 1)$, where m is the slope (equivalently the tangent of the angle the dashed line makes where it cuts the x-axis). So the problem is to find this tangent.

Let β be the angle by which the line $x + y - 2 = 0$ cuts the the x-axis. Since this line has slope -1, $\tan(\beta) = -1$. From the left hand figure, $\tan(\alpha) = \frac{1}{3}$. The dashed line cuts the x-axis with an angle of $\gamma = \alpha + \beta$, so

$$\tan(\gamma) = \tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} = \frac{(\frac{1}{3}) + (-1)}{1 - (\frac{1}{3})(-1)} = \frac{-\frac{2}{3}}{\frac{4}{3}} = -\frac{1}{2}.$$

Thus $m = -\frac{1}{2}$, which shows that the desired equation is $y - 1 = -\frac{1}{2}(x - 1)$. This can be scaled and rewritten as $x + 2y - 3 = 0$.