Logic.

The fact that $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous function is sometimes written

$$\forall x_0\ \forall \varepsilon\ \exists \delta\ \forall x\ ((\varepsilon \le 0) \vee ((0 < |x - x_0| < \delta) \to (|f(x) - f(x_0)| < \varepsilon)))$$

We will learn how to read and write such sentences, and how to test whether a sentence like this is true.

We will discuss the following four topics.

- **Syntax.** (What are the accepted ways to combine symbols into sensible expressions?)
- **Meaning.** (What does it mean for an expression to be true or false?)
- **Proof.** (How can we determine truth or communicate it to others?)
- **The relationship between truth and provability.** (Are all provable statements true? Are all true statements provable?)

(I) Syntax.
- (1) An alphabet of symbols.
  - (a) variables: $x_1, x_2, x_3, \ldots$
  - (b) logical symbols
    - (i) equality: $=$
    - (ii) logical connectives: $\wedge, \vee, \neg, \to, \leftrightarrow$.
    - (iii) quantifiers: $\exists, \forall$
  - (c) nonlogical symbols: operation symbols, predicate symbols
  - (d) punctuation: parentheses, commas.
- (2) Formulas and sentences.

(II) Meaning.
- (1) The logical connectives.
- (2) The equivalence of $(H \to C)$, $((\neg C) \to (\neg H))$ and $((H \wedge \neg C) \to \text{False})$.
- (3) Structures.
- (4) Valuations.
- (5) The symbol $\models$
- (6) The equivalence of $\neg(\exists x P)$ and $\forall x(\neg P)$.

(III) Proof.
- (1) Axioms.
- (2) Rules of deduction.
- (3) The symbol $\vdash$

(IV) The relationship between truth and provability.