Euclidean and Non-Euclidean Geometry (MATH 3210):
some review problems

(1) How do you define points, line, incidence, betweenness and congruence in a Cartesian plane?

(2) Find one of Hilbert's axioms that fails in the Cartesian plane over the field of rational numbers.

(3) Explain why $\sqrt[3]{2}$ is not a constructible number.

(4) Show that $\sqrt[4]{2} \in K$, but $\sqrt[4]{2} \notin \Omega$.

(5) Which of the following straightedge and compass construction problems are solvable?
   (a) Cubing the sphere? (The 3-dimensional version of squaring the circle.)
   (b) Doubling the 4-dimensional cube? (That is, given the side length of a 4-dimensional cube, construct the side length of a 4-dimensional cube of twice the volume.)
   (c) Constructing a regular 30-gon?
   (d) Constructing a golden rectangle? (That is, an $a \times b$-rectangle where $(a+b)/a = a/b$.)

(6) Let Π be the Dehn plane. Explain why Playfair's postulate fails for every choice of a line ℓ and a point A not on ℓ. (That is, show that given any ℓ and A with A not on ℓ, there are multiple parallels to ℓ through A.)

(7) What is the definition of "complete ordered field"? Why must a complete ordered field be Archimedean?

(8) How do you define points, line, incidence, betweenness and congruence in the Poincare disk model?

(9) Explain how to construct limiting parallels in the Poincare disk model. Explain why the Dehn plane does not have limiting parallels.

(10) Explain why the hyperbolic center of a circle in the Poincare model is not the same as the Cartesian center.