Euclidean and Non-Euclidean Geometry (MATH 3210): REVIEW SHEET 2

From the book: pages 96–147, 158–162, 186–191, 242–247, 295–300, 334–342, 355–366.

V. Cartesian planes
   (a) Cartesian planes over ordered fields, Pythagorean ordered fields, and Euclidean ordered fields.
   (b) Cartesian planes satisfy Playfair's postulate.
   (c) Cartesian planes are semi-Euclidean.
   (d) Hilbert's axioms do not imply the line-circle intersection property or the circle-circle intersection property.

VI. Straightedge and compass construction problems
   (a) A point in the real Cartesian plane is constructible from the empty configuration if and only if its coordinates lie in the field of real constructible numbers.
   (b) A real number is constructible if and only if the Galois group of its minimal polynomial has degree that is a power of 2.
   (c) Some circles can be squared, but the general circle cannot be squared.
   (d) Some cubes can be doubled, but the general cube cannot be doubled.
   (e) Some angles can be trisected, but the general angle cannot be trisected.
   (f) Some regular n-gons can be constructed, but some cannot be.

VII. Alternative axioms related to Playfair's postulate.
   (a) Archimedes axiom.
   (b) Dedekind's completeness axiom.

VIII. Non-Euclidean geometry
   (a) The Dehn plane: a non-Archimedean example.
   (b) Circular inversion. The Poincare model.

General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

(i) Know the definitions of new concepts, and the meanings of the definitions.
(ii) Know the statements and meanings of the major theorems.
(iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
(iv) Know how to perform the different kinds of calculations discussed in class.
(v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
(vi) Know how to correct mistakes made on old HW.