

## Euclidean and Non-Euclidean Geometry (MATH 3210): REVIEW SHEET

From the book: pages 1–96.

- I. Euclid's geometry.
  - (a) The structure of the Elements: definitions, postulates, common notions, propositions.
  - (b) Be able to state Euclid's postulates in your own words.
  - (c) Ruler and compass constructions (be able to say what is allowed).
- II. Logic: models.
  - (a) Be able to define what kind of structure a plane geometry is:  $\Pi = \langle \mathcal{P}, \mathcal{L}; I, B, \overline{C}, C_{\triangle} \rangle$ .
  - (b) Models.
  - (c) Truth and satisfaction. The double turnstyle symbol  $\models$ .
  - (d) Isomorphism. Isomorphism invariant.
  - (e) Know examples of models of some sets of axioms (small models, Cartesian planes over different fields, disk model).
- III. Logic: syntax.
  - (a) Axioms. Definitions.
  - (b) Provability. The single turnstyle symbol  $\vdash$ .
  - (c) First-order statements and Gödel's Completeness Theorem.
  - (d) Independence of statements. Completeness and consistency. Categoricity.
- IV. Hilbert's axioms.
  - (a) Three incidence axioms. (Plus definition of projective plane.)
  - (b) Four betweenness axioms. (Plus plane and line separation, Crossbar Theorem.)
  - (c) Three axioms of segment congruence.
  - (d) Three axioms of angle congruence.
  - (e) Parallel lines, segment, endpoints, triangle, ray, angle, vertex, interior, circle, supplementary angles, vertical angles, right angles.

### General advice on preparing for a math test.

Be prepared to demonstrate understanding in the following ways.

- (i) Know the definitions of new concepts, and the meanings of the definitions.
- (ii) Know the statements and meanings of the major theorems.
- (iii) Know examples/counterexamples. (The purpose of an example is to illustrate the extent of a definition or theorem. The purpose of a counterexample is to indicate the limits of a definition or theorem.)
- (iv) Know how to perform the different kinds of calculations discussed in class.
- (v) Be prepared to prove elementary statements. (Understanding the proofs done in class is the best preparation for this.)
- (vi) Know how to correct mistakes made on old HW.