Geometry
Quiz 6

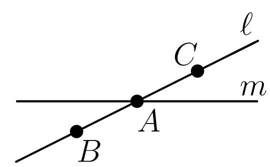
Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

Please read the following Theorem and "Proof".

Theorem. If ℓ and m are parallel, then all points of ℓ lie on the same side of m.

"Proof". Assume that ℓ and m are not parallel. Let A be a point incident to both lines. By $I2$, there is point $B \neq A$ also on ℓ. By $B2$, there is a point C such that $B * A * C$. Now B and C are on ℓ, but are not on the same side of m. $\square$



(1) Explain why the "proof" does not prove the theorem.

The Theorem statement has the form $A \to B$ where A is "ℓ and m are parallel" and B is "all points of ℓ lie on the same side of m". The proof (correctly) argues that $(\neg A) \to (\neg B)$, which is the inverse statement.Typically statements are not equivalent to their inverses, so the proof does not establish the Theorem.

(2) Give a correct proof.

This is a proof by contradiction, so assume that ℓ and m are parallel, but that there are points $A \neq B$ that are not on the same side of m. Neither A nor B lies on m, since ℓ and m have no common points, so A and B lie on opposite sides of m. This means that there is a point C incident to m such that $A * C * B$ holds. Now $B1$ implies that C is on ℓ, so it is a point common to ℓ and m. This contradicts the assumption that ℓ and m are parallel.