Practice.

This exercise concerns perpendiculars and parallels. (Some hints are on the back.)

(1) Explain why no triangle can have two or more right angles.

(2) Explain why if ℓ is a line that meets distinct lines m and n in right angles, then m and n are parallel.

(3) Explain why if ℓ is a line and A is any point (possibly on ℓ, possibly not), then there is a line m incident to A that meets ℓ in a right angle.

(4) Suppose that A is not incident to ℓ. Explain how to construct a line parallel to ℓ through A.

Hints:

(1) Use the Exterior Angle Theorem (Proposition 10.3). See what contradiction you get if $\angle ACB$ and $\angle CAB$ are right angles.

(2) Assume not. Show that you get a triangle with at least two right angles.

(3) (Case where A is not incident to ℓ.) Pick points $B \neq C$ on ℓ and observe that ABC is a triangle. Now reflect ABC through ℓ, so that $ABC \cong A'BC$ where A' is chosen on the side of ℓ opposite A. Explain why the line $m = AA'$ works.

(Case where A is incident to ℓ.) Construct a right angle somewhere and copy it along ℓ on a ray emanating from A.

(4) Construct a perpendicular to a perpendicular.