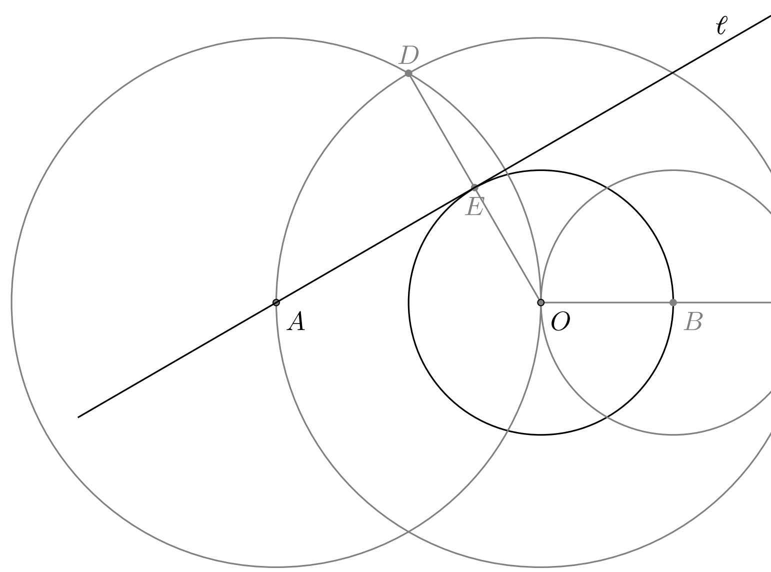
Homework 1

Mohammad Ozaslan Euclidean and Noneuclidean Geometry February 1, 2017

Section 2

2.8. Given a circle with center O and a point A outside the circle. Choose B on $\odot O$ at random.

1. Draw $\odot BO$.
2. Construct and extend BO, get C.
3. Construct $\odot OC$.
4. Construct $\odot AO$, get D.
5. Draw DO, get E.
6. Draw a tangent line AE, or ℓ.



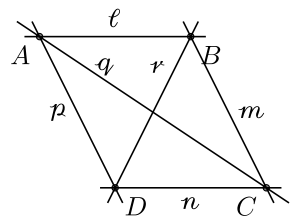
Section 6

6.1. Let $\mathscr{P} = \{A, B, C, D\}$, $\mathscr{L}_1 = \{\ell, m, n, p, q, r\}$, and $\mathscr{L}_2 = \{\ell, m, n, p\}$.

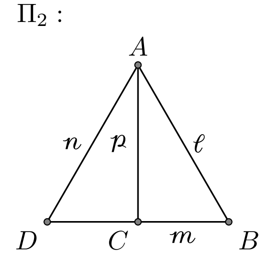
Let $I_1 = \{(A, \ell), (A, q), (A, p), (B, \ell), (B, r), (B, m), (C, m), (C, q), (C, n), (D, n), (D, r), (D, p)\}$ and let $I_2 = \{(A, \ell), (A, p), (A, n), (B, \ell), (B, m), (C, m), (C, p), (D, m), (D, n)\}$.

Then, $\Pi_1 = \langle \mathscr{P}, \mathscr{L}_1 \mid I_1 \rangle$ and $\Pi_2 = \langle \mathscr{P}, \mathscr{L}_2 \mid I_2 \rangle$ are the only incidence geometries (up to isomorphism) with four points, and are modeled below:

Π_1 :



Π_2 :



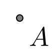
Π_1 and Π_2 both satisfy (P).

6.3. A *projective plane* is a set of points and subsets called lines that satisfy the following four axioms:

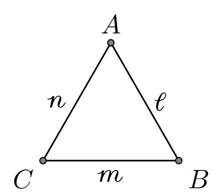
P1. Any two distinct points lie on a unique line.
P2. Any two lines meet in at least one point.
P3. Every line contains at least three points.
P4. There exist three noncollinear points.

(c) The axioms (P1), (P2), (P3), (P4) are independent.

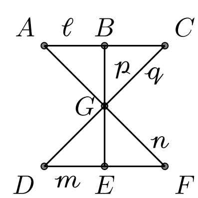
Proof.
$(P1 \wedge P2 \wedge P3 \wedge \neg P4)$
Let $\mathscr{P} = \{A\}$, and let $\mathscr{L} = \varnothing = I$.
Then, $\Pi = \langle \mathscr{P}, \mathscr{L} \mid I \rangle$ is a plane satisfying all of the axioms above except P4:



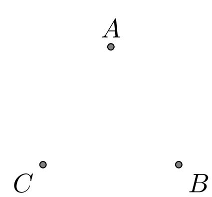
$(P1 \wedge P2 \wedge \neg P3 \wedge P4)$
Let $\mathscr{P} = \{A, B, C\}$, and $\mathscr{L} = \{\ell, m, n\}$.
Then, let $I = \{(A,\ell), (A,n), (B,\ell), (B,m), (C,m), (C,n)\}$.
Thus, $\Pi = \langle \mathscr{P}, \mathscr{L} \mid I \rangle$ is a plane satisfying all of the axioms above except P3:



$(P1 \wedge \neg P2 \wedge P3 \wedge P4)$
Let $\mathscr{P} = \{A, B, C, D, E, F\}$, and $\mathscr{L} = \{\ell, m, p, q\}$.
Then, let $I = \{(A,\ell), (B,\ell), (C,\ell), (D,m), (E,m), (F,m), (G,n), (G,p), (G,q)$ $(A,n), (B,p), (C,q), (D,q), (E,p), (F,n)\}$.
Thus, $\Pi = \langle \mathscr{P}, \mathscr{L} \mid I \rangle$ is a plane satisfying all of the axioms above except P2:



$(\neg P1 \wedge P2 \wedge P3 \wedge P4)$
Let $\mathscr{P} = \{A, B, C\}$ and $\mathscr{L} = I = \varnothing$.
Then, $\Pi = \langle \mathscr{P}, \mathscr{L} \mid I \rangle$ is a plane satisfying all of the axioms above except for P1:



We have shown that any three statements do not necessarily imply the fourth. Thus, all of these axioms are independent. □