Set Theory
Quiz 8

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Assume that $\mathbb{T} = \langle T; < \rangle$ is a totally-ordered set and $\mathbb{W} = \langle W; < \rangle$ is a well-ordered set. Show that if there is a morphism $f \colon \mathbb{T} \to \mathbb{W}$, then $\mathbb{T}$ must also be a well-ordered set. ("morphism" means $<$-preserving function.)

We must show that if $S \subseteq T$ is nonempty, then S has a $<_T$-least element in the order of $\mathbb{T}$. (To avoid confusion, I write $<_T$ for the order relation of $\mathbb{T}$ and $<_W$ for the order relation of $\mathbb{W}$.)

Since $S \neq \emptyset$ we conclude that $f(S)$ is nonempty. Since $\mathbb{W}$ is a well-order and $f(S) \subseteq W$ is nonempty, there is a $<_W$-least element $f(s) \in f(S)$. We claim that s is a $<_T$-least element of S. If this were not so, then there would be some $s' \in S$ with $s' <_T s$. Then $f(s') <_W f(s)$ (since f is a morphism), contradicting the choice of s.