

Set Theory
Quiz 7

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Assuming the axioms of ZFC, explain why $(b) \Rightarrow (a)$.¹

(a) $|A| \leq |B|$.

(b) Either $A = \emptyset$ or there is a surjective function $g: B \rightarrow A$.

If $A = \emptyset$, then the empty function $\emptyset: A \rightarrow B$ witnesses that $|A| \leq |B|$.

Assume that $A \neq \emptyset$ and $f: B \rightarrow A$ is surjective. The Axiom of Choice guarantees the existence of a section $s: A \rightarrow B$ to f . Sections to surjective functions are injective, so s witnesses that $|A| \leq |B|$.

¹The implication $(a) \Rightarrow (b)$ also holds, but you don't have to prove it.