

Set Theory
Quiz 5

Name: _____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Show that if X is finite, then $X \cup \{a\}$ is finite.

Assume that X is finite. This means that there exists some $k \in \mathbb{N}$ and some bijection $f: k \rightarrow X$.

Case 1. ($a \in X$)

In this case, $f: k \rightarrow X = X \cup \{a\}$ is a bijection. Since $k \in \mathbb{N}$, $X \cup \{a\}$ is finite.

Case 2. ($a \notin X$)

We seek a bijection $\hat{f}: S(k) \rightarrow X \cup \{a\}$. We define $\hat{f} = f \cup \{(k, a)\}$. The facts that (i) f is a bijection from k to X , (ii) $k \in S(k) - k$, (iii) $a \in (X \cup \{a\}) - X$, and (iv) $\hat{f} := f \cup \{(k, a)\}$ suffice to guarantee that $\hat{f}: S(k) \rightarrow X \cup \{a\}$ is a bijection. Since $S(k) \in \mathbb{N}$, $X \cup \{a\}$ is finite.