Set Theory
Quiz 4

Name:_____

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Prove that addition is cancellative: that is, for all m, n, k in $\mathbb{N}$ we have

$$m + k = n + k \quad \text{implies} \quad m = n.$$

(If you need some lemmas in the course of the proof, be sure to state them. You do not need to prove the lemmas now.)

Let $\varphi(x)$ be: "$\forall m \forall n((m + x = n + x) \to (m = n))$"

Basis of Induction: We prove that $\varphi(0)$ holds. Assume that the premise of the implication in $\varphi(0)$ is true, namely that $m + 0 = n + 0$ holds for some m, n.

$$\begin{aligned} m &= m + 0 && (\text{IC}, +) \\ &= n + 0 && \text{Assumption} \\ &= n && (\text{IC}, +) \end{aligned}$$

Inductive Step: Assume that $\varphi(k)$ holds and that the premise $m + S(k) = n + S(k)$ of the implication in $\varphi(S(k))$ holds for some m, n.

$$\begin{aligned} m + S(k) &= n + S(k) && \text{Premise of } \varphi(S(k)) \\ S(m + k) &= S(n + k) && (\text{RR}, +) \\ m + k &= n + k && \text{Successor is injective} \\ m &= n && \text{Inductive Hypothesis} \end{aligned}$$

Hence $\varphi(S(k))$ holds.