Set Theory
Quiz 11

Name:______________________________

You have 10 minutes to complete this quiz. If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show your work, and give adequate explanations.

1. Explain why $\text{rank}(\omega) = \omega$.

The rank of a set x is the least ordinal α such that $x \subseteq V_\alpha$ (or, equivalently, $x \in V_{\alpha+1}$).

We remarked in class that $\text{rank}(\alpha) = \alpha$ for any ordinal α, so $\text{rank}(\omega) = \omega$.

2. Let $<$ be the order relation on ω. What is $\text{rank}(<)$?
$\text{rank}(<) = \omega$.

Reasoning: The rank of $<$ is the least ordinal α such that V_α contains each of the elements of the relation $<$. An element of $<$ has the form (m, n) with $m, n \in \omega$ and $m < n$. This is a set of the form $\{\{m\}, \{m, n\}\}$ with $m < n$. Since $m \subseteq n \subseteq V_n$ we have $m, n \in V_{n+1}$, so $\{m\}, \{m, n\} \in V_{n+2}$, so $\{\{m\}, \{m, n\}\} \in V_{n+3} \subseteq V_\omega$. This shows that V_ω contains any pair of $<$. This is enough to show that $\text{rank}(<)$ is at most ω. To show that $\text{rank}(<)$ cannot be strictly smaller than ω, note that $<$ cannot be a subset of V_k for finite k, since $<$ is an infinite set and V_k is a finite set.