1. (Exercise 7.1.3) For any set A, there is a mapping of $\mathcal{P}(A \times A)$ onto $h(A)$.

Define $f : \mathcal{P}(A \times A) \to h(A)$ by

$$f(R) = \begin{cases} \alpha & \text{if } R \text{ is a well-ordering of type } \alpha \text{ of a subset of } A \\ 0 & \text{otherwise.} \end{cases}$$

There exists an $R \subseteq A \times A$ that is a well-ordering of type α of a subset of A iff α is embeddable in A iff $\alpha < h(A)$. Thus, f maps onto $h(A)$.

2. (Exercise 7.1.4.) $|A| < |A| + h(A)$. (It suffices to explain why $|A| < |A \cup h(A)|$.)

Let's prove the second version of the problem. The inclusion function $\iota : A \to A \cup h(A)$ is 1-1, so $|A| \leq |A \cup h(A)|$. If we had equality, $|A| = |A \cup h(A)|$, then there would exist a bijection $f : A \cup h(A) \to A$. The restriction of f to $h(A)$ would be an embedding of $h(A)$ into A. This is impossible, so $|A| \neq |A \cup h(A)|$, and therefore $|A| < |A \cup h(A)|$.

The original version of the problem follows from the second version as follows. Let A' be a set equipotent with A and which contains no ordinals. Then $|A'| = |A|$ and $h(A') = h(A)$, but we have arranged that the sets A' and $h(A')$ are disjoint. Now

$$|A| = |A'| < |A' \cup h(A')| = |A'| + |h(A')| = |A'| + h(A') = |A| + h(A).$$

3. (Exercise 7.1.5.) $|h(A)| < |\mathcal{P}(\mathcal{P}(A \times A))|$ for all A.

It is a general fact that if $f : X \twoheadrightarrow Y$ is surjective, then $f^{-1} \colon Y \rightarrowtail \mathcal{P}(X) : y \mapsto f^{-1}(y)$ is injective. Applying this to $f : \mathcal{P}(A \times A) \twoheadrightarrow h(A)$ from Problem 7.1.3 we get that $f^{-1} : h(A) \rightarrowtail \mathcal{P}(\mathcal{P}(A \times A))$ is 1-1.