(1) (Exercise 1.3.1.) Show that the set of all x such that $x \in A$ and $x \notin B$ exists.

The set of the problem, which is typically denoted by $A - B$ or $A \setminus B$, may be constructed by the Axiom of Separation as follows:

$$A - B = \{x \in A \mid x \notin B\}.$$

(2) (Exercise 1.3.6.) Show that $\mathcal{P}(X) \subseteq X$ is false for any X. In particular, $\mathcal{P}(X) \neq X$ for any X. This proves again that a "set of all sets" does not exist. [Hint: Let $Y = \{u \in X \mid u \notin u\}$; $Y \in \mathcal{P}(X)$ but $Y \notin X$.]

Assume that $\mathcal{P}(X) \subseteq X$ and let $Y = \{u \in X \mid u \notin u\}$. We must have $Y \subseteq X$, so $Y \in \mathcal{P}(X)$, so we should have $Y \in X$. Let us show that the assumption that $Y \in X$ leads to a Russell-type paradox.

Case 1: ($Y \in X$ and $Y \notin Y$) Then $u := Y$ satisfies the property that defines membership in Y, so $Y = u \in Y$. The properties $Y \notin Y$ and $Y \in Y$ contradict one another.

Case 2: ($Y \in X$ and $Y \in Y$) Then $u := Y$ fails the property that defines membership in Y, so $Y = u \notin Y$. The properties $Y \in Y$ and $Y \notin Y$ contradict one another.

(3) (a) (Exercise 2.2.8(a)) $A \times B = \emptyset$ if and only if $A = \emptyset$ or $B = \emptyset$.

[$\Rightarrow$] : Arguing the contrapositive, assume that $A \neq \emptyset \neq B$. There must exist $a \in A$ and $b \in B$, in which case $(a, b) \in A \times B$, so $A \times B \neq \emptyset$.

[$\Leftarrow$] : Again, arguing the contrapositive, assume that $A \times B \neq \emptyset$. There must exist $(a, b) \in A \times B$, in which case $a \in A$ and $b \in B$, so $A \neq \emptyset$ and $B \neq \emptyset$.

(3) (b) (First part of Exercise 2.2.8(b)) $(A_1 \cup A_2) \times B = (A_1 \times B) \cup (A_2 \times B)$.

Choose an element (a, b) from the left hand side: $(a, b) \in (A_1 \cup A_2) \times B$. By the definitions of $\times$ and $\cup$, we have $b \in B$ and either $a \in A_1$ (Case 1) or $a \in A_2$ (Case 2). If we are in Case 1, then $(a, b) \in A_1 \times B$; if we are in Case 1, then $(a, b) \in A_2 \times B$. In either case we have $(a, b) \in (A_1 \times B) \cup (A_2 \times B)$, according to the definition of $\cup$.

Now choose an element (a, b) from the right hand side: $(a, b) \in (A_1 \times B) \cup (A_2 \times B)$. By the definition of $\cup$, we have $(a, b) \in A_1 \times B$ (Case 1) or $(a, b) \in A_2 \times B$ (Case 2). This yields $b \in B$ in either case and either $a \in A_1$ (in Case 1) or $a \in A_2$ (in Case 2). Therefore, in either case we have $a \in A_1 \cup A_2$ and $b \in B$, so $(a, b) \in (A_1 \cup A_2) \times B$.