Stirling numbers and Bell numbers

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There are many parallels between C(n, k) and S(n, k).

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where $x^{\underline{k}} = (x)_{k} = x(x-1) \cdots (x-(k-1)).$

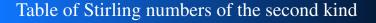


Table of Stirling numbers of the second kind

$n \setminus k$	0	1	2	3	4	5	6	7	8	
0	1	0	0	0	0	0	0	0	0	
1	0	1	0	0	0	0	0	0	0	
2	0	1	1	0	0	0	0	0	0	• • • •
3	0	1	3	1	0	0	0	0	0	• • • •
4	0	1	7	6	1	0	0	0	0	
5	0	1	15	25	10	1	0	0	0	
6	0	1	31	90	65	15	1	0	0	
7	0	1	63	301	350	140	21	1	0	
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The *n* row sum is denoted B_n and is called the *n*th **Bell number**.

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$$035/14/2 \longrightarrow 53\underline{0}4\underline{1}\underline{2}$$

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- $n! \le n^n$, since the latter counts the number of functions $f: n \to n$, while the former only counts the bijections.
- $n^n \le 2^{n^2}$, since the latter counts the number of binary relations from n to n, while the former only counts the binary relations that are functions.