A key part of doing mathematics is communicating mathematics - both in person and in writing. Here we'll talk about what "If/Then" statements do and do not mean, and how to use them in your mathematical writing.

The If/Then Statement:

The "If/Then" statement is used to show *implication*. "If (Statement A), then (Statement B)" can also be phrased as "(Statement A) implies (Statement B)". This means that if Statement A is true then Statement B must also be true.
Importantly, this does not mean that if Statement B is true, then Statement A is also true.
Another way to think about this is that Statement A is a condition (or list of conditions) to check before we can conclude that Statement B is true.

Examples:

- **If your pet is a goldfish, then your pet has scales.**

  This doesn't mean that if your pet has scales then your pet must be a goldfish. Perhaps you have a dragon or an iguana.

- **Duck's Theorem: If your pet has webbed feet and can fly, then your pet is a bird.**

  Here we have a list of things to check – in order to conclude your pet is a bird, we need to check that your pet can fly AND that your pet has webbed feet. (Otherwise, it might be a platypus or a bumblebee.)

  If your pet is a bird, does Duck's Theorem say that your pet must have webbed feet or the ability to fly?

- **If x is prime, then x is odd.**

  What would the opposite statement be? Is it true?

- **Comparison Test for Integrals:**

  If f, g are integrable on $[a, \infty)$, $f(x) \geq g(x)$ for all $x \geq a$ and $\int_a^\infty f(x)\ dx$ converges, then $\int_a^\infty g(x)\ dx$ converges.

  What things need to be checked before we can conclude $\int_a^\infty g(x)\ dx$ converges?

  If the conclusion is true $\left(\int_a^\infty g(x)\ dx \text{ converges}\right)$, does $\int_a^\infty f(x)\ dx$ also converge?

Using If/Then Statements in a Written Argument:

To use the conclusion of an If/Then statement, you first need to show the reader that the "If" conditions are satisfied. Make sure you thoroughly explain how you know the conditions are met. If your "If/Then" statement has a name (such as the Intermediate Value Theorem or the p-test) then be sure to include that name.

Writing Examples

- "Since my pet is a fish, my pet has scales."
- "My family's pet Arwin has webbed feet. Also, Arwin can fly. So, by Ducky's Theorem, Arwin is a bird."
- "Lana's pet can fly, but doesn't have webbed feet. So, Ducky's Theorem doesn't tell us anything about Lana's pet."
- "The only factors of 13 are 1 and 13, so 13 is prime. Therefore, I know that 13 is odd."
- "By the p-test, $\int_1^\infty \frac{1}{x^5}\,dx$ converges, since $p = 5$ and $5 \geq 1$. For all $x \geq 1$, $\frac{1}{x^5+10} \leq \frac{1}{x^5}\,dx$ and $\frac{1}{x^5+10}, \frac{1}{x^5}$ are integrable on $[1, \infty)$. Therefore, the Comparison Test for Integrals tells us $\int_1^\infty \frac{1}{x^5+10}\,dx$ converges. "
- "The functions $\frac{1}{x}$ and $\frac{1}{2x}$ are integrable on $[5, \infty)$, and $\int_5^\infty \frac{1}{x}\,dx$ diverges, which we showed in problem 16. Since $\frac{1}{x} \leq \frac{1}{2x}$ for all $x \geq 5$, $\int_5^\infty \frac{1}{2x}\,dx$ diverges by the Comparison Test for Integrals."

  Note: This only works if you actually did show that $\int_5^\infty \frac{1}{x}\,dx$ diverges in problem 16.