Objectives:

- Write Riemann sums efficiently.
- Identify when a sum is an over or underestimate of area.

Motivation: Suppose we wanted to estimate the area between $f(x) = x^2$ and the x-axis from $x = 0$ to $x = 200$ using 100 rectangles. We know how to do this:

However, there must be a shorter way to write this.

Sigma Notation: The symbol _____ , pronounced _____ , stands for _____ .

Examples of sigma notation:

1. Add up all numbers from 1 to 100:

2. Add up all the squares of nonnegative integers less than 10.

3. Add up five variables labeled $x_3, x_4, \ldots, x_7$.

Using Sigma Notation for Riemann Sums

Suppose we want to come up with an estimate of the area between $f(x)$ and the x-axis between $x = a$ and $x = b$ using n rectangles. We know how to do this using a Riemann sum.

In order to put our sum in Sigma notation, we need to give the rectangle endpoints names.

The width of each rectangle is written as Δx. To find the height of the rectangle, we need to choose a point in each smaller interval:

So our area estimate is given by:

If we take a right hand sum, $x_i^* =$ _____ . How can we express x_i^* in terms of known values?

So the value of x_i depends on Δx. How can we express Δx in terms of known values?

Example Estimate the area between $g(x) = \tan(x^2)$ and the x-axis from $x = -\pi/4$ to $x = \pi/4$ using $n = 10$ intervals.

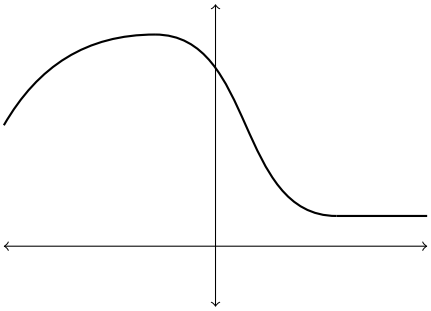
We can increase the accuracy of this estimate by _____ .

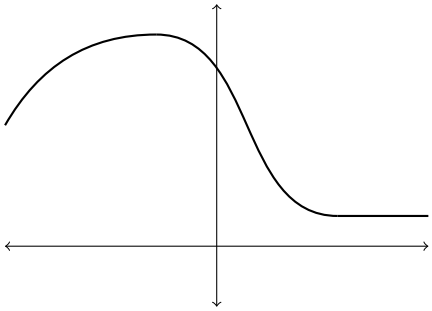
The exact value of the area can be found by _____ .

Write an expression for the exact area in this example. (Do not evaluate the limit.)

Over and Underestimates:

Sketch 5 to 10 rectangles for each estimate. Use your picture to fill in the chart.





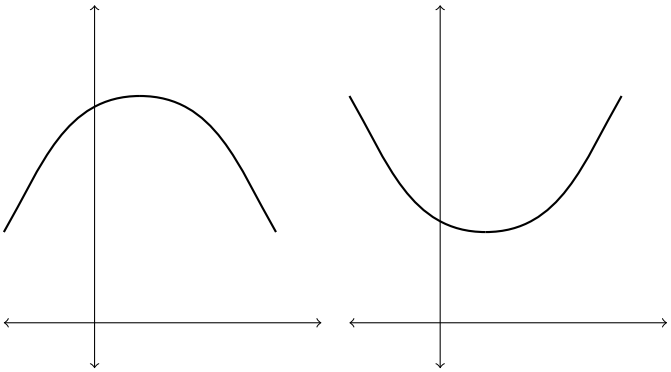
Left Hand Estimate

When f is:	LH estimate is:
Constant	
Increasing	
Decreasing	

Right Hand Estimate

When f is:	RH estimate is:
Constant	
Increasing	
Decreasing	

To determine if the **Trapezoidal** rule is an under or overestimate, we need to look at concavity: (These ones will be easier to see if you use very large trapezoids - only 2-3 trapezoids per curve.)



When f is:	Trap estimate is:
Concave down	
Concave up	