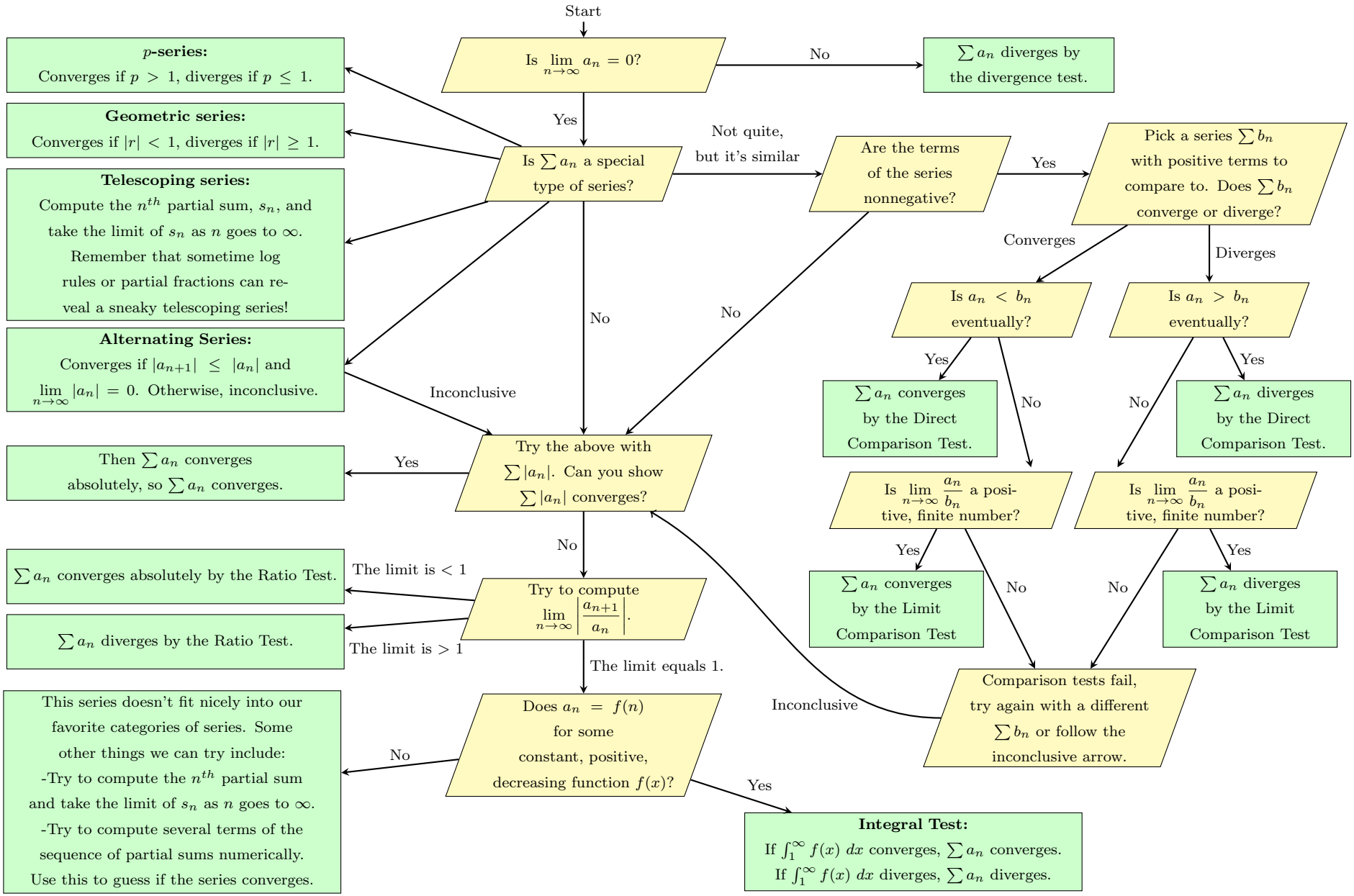
Let $\sum a_n$ be an infinite series. This flowchart is useful for deciding which convergence/divergence test to use.

We cannot include every single possibility. This chart is meant to be a useful guide, but it is no substitute for your own understanding and critical thinking!



Other Strategies

Always try the Test for Divergence First!

If the terms of the series don't go to zero, there's no need to try any other test.

Guess if the series converges

Having a hunch about the convergence of the series can inform your strategy. For example, if you think a series converges, when you try the Direct Comparison Test you know to look for a convergent series where the terms are larger than the terms of the given series. Whereas if you think the series diverges, you know to look for a divergent series with smaller terms.

Only try tests you're allowed to use!

There's no point in trying to use a test if the series doesn't fit the necessary hypotheses. For example, if the terms of a series aren't positive, that auromatically rules out the Direct Comparison Test, Limit Comparison Test, and Integral Test.

Remember a test can be inconclusive

Some series will fail some convergence tests—be careful not to conclude that the series automatically diverges! If the test is inconclusive, we need to try a different test.

Rewrite the series terms

If the series is uncooperative try to simplify or rewrite fractions—you can even use a partial fractions decomposition. Series with logarithms of quotients or products can often be rewritten with log rules.

Know your nice examples

Some tests play especially nicely with certain series.

- Exponential and factorial terms often work well with the Ratio Test.
- Rational functions with roots and polynomials are often worth applying the Direct or Limit Comparison Test to.
- If $a_n = f(n)$ for a nice-to-integrate function $f(x)$, that's a good hint to try the Integral Test. (Don't forget to check to see if $f(x)$ is positive, continuous, and decreasing on the domain you need.)

Test for absolute convergence first

If you're asked to find whether $\sum a_n$ converges absolutely or conditionally, you will have to determine whether $\sum |a_n|$ eventually, so you may as well start with it. Then if $\sum |a_n|$ converges you don't need to separately check convergence of $\sum a_n$.

Write out a few terms of the series

Write out both the first few terms of the series and the first few terms of the sequence of partial sums. This can help you get a feel for the series.

Don't Give Up!

There are many series tests for a reason - not all tests work for all series. Persistence is a necessary tool in your series toolbox.