Objectives:

- Interpret average and instantaneous rates of change in real world contexts.

Rates of Change

Both average and instantaneous rates of change are slopes - "change in y over change in x", so the units for rates of change are $\frac{\text{y-units}}{\text{x-units}}$.

If you're unsure of the meaning of a function's rate of change, try substituting the appropriate labels or units into some of the following phrases:

- "Change in y-values over x-values"
- "Increase/decrease in y-values as x-values increases" "Increase/decrease in y-units per x-unit"
- "Increase/decrease in y-units per x-unit" or "y-units per x-units"

For example, if x is time in hours and $f(x)$ is distance in miles, the phrase "miles per hour" demonstrates that the rate of change is speed.
If x is number of units produced and $f(x)$ is total profit in dollars, "change in profit over units" is a little puzzling, but "increase in profit per unit produced" is more helpful.

Complete the table. (Come up with your own two functions for the last two rows, or use other examples you encounter in your textbook or WebAssign exercises.)

x-values	y-values	Derivative at a (Instantaneous Rate of Change)
Time (Hours)	Distance (Miles)	$\frac{\Delta\text{Miles}}{\Delta\text{Hours}}$=velocity in miles per hour at a hours.
Time (Seconds)	Distance (Feet)	
Units sold (number)	Total revenue ($)	
Time (weeks)	Height of plant (inches)	
Radius of a circle (cm)	Area of the circle (cm^2)	

Example 1
Choose appropriate units in which to measure your height, $f(x)$ = height in ________________
and units for measuring time (from when you were born until now), x = time in ____________

Estimate $f(0)$ and $f(a)$ where a is the time now.

What is the average rate of change between $x = 0$ and $x = a$? (Include units.)

Estimate $f'(a)$. (Include units.)

Is the instantaneous rate of change at $x = a$ the same as the average rate of change? Why or why not?

Example 2 The following table gives the population of a town, p, as a function of the year. Estimate $p'(2012)$ by calculating an average rate of change. Include units! What does $p'(2012)$ represent?

Year	2010	2012	2014
Population	10,000	11,150	13,360

Challenge:
The function $A(r) = \pi r^2$ gives the area of a circle $\left(\text{cm}^2\right)$ as a function of its radius, r (cm).
Use the limit definition of a derivative to find $A'(a)$ for an arbitrary radius $r = a$.
What does this derivative mean?