Important Note:
This is NOT a thorough review for the exam. It contains only one question from each book section to help you identify areas you should concentrate on while studying. Many more study materials can be found on the course webpage under the "Exams" tab.

1. Find any absolute and local extrema of $f(x) = x\sqrt{x+3}$ on $(-3, \infty)$

   The domain of our function is $[-3, \infty)$.
   1 point - Find $f'(x) = \sqrt{x+3} + \dfrac{x}{2\sqrt{x+3}}$
   2 point - Find critical points of f (where f' is 0 or undefined): $x = -3$ (not in domain) and $x = -2$.
   2 point - Use first or second derivative test to find that $f(x)$ has a local minimum of $(-2)\sqrt{-2+3} = -2$ at $x = -2$.
   (Don't forget to label your sign chart if you use first derivative test!)
   1 point - Write a sentence explaining why the test you used indicates there is a local min at $x = -2$.
   2 points - Write a sentence explaining that $(-2, -2)$ is an absolute maximum on the given interval because it is the only critical point in the interval and it is a local minimum.

   More practice for this section:
   (1) What would you do if you were asked to find the extrema of $f(x)$ on the interval $[-2, 2]$? How would you explain why you can use this method?
   (2) How would you find any inflection points of $f(x)$? Make sure to justify your answer.
   (3) Using this information, sketch a graph of $f(x)$.

2. Remember that the marginal cost is the rate of change of cost. Suppose it costs 1 million dollars to produce 1,000 hoverboards and 2 million dollars to produce 3,000 hoverboards. What assumptions do we need to make in order to conclude that the marginal cost is \$500 per hoverboard for some number of hoverboards between 1,000 and 3,000. Name any theorems you use.

   Let $C(x)$ be the cost to produce x hoverboards. Then marginal cost is given by $C'(x)$. The information in the problem tells us $C(1,000) = 1,000,000$ and $C(2,000) = 2,000,000$.
   2 pt - Find average marginal cost between $x = 1,000$ and $x = 3,000$: $\dfrac{C(3,000) - C(1,000)}{3,000 - 1,000} = \dfrac{1,000,000}{2,000} = 500\dfrac{\$}{\text{hoverboard}}$.
   2 pt - State that if the hypotheses of the Mean Value Theorem are satisfied, then $C'(a) = 500$ for some $1,000 < a < 3,000$.
   2 pt - Conclude that the necessary assumptions are (1) $C(x)$ is continuous on $[1,000, 3,000]$ and (2) $C(x)$ is differentiable on $(1,000, 3,000)$.

3. Suppose the hoverboard factory from before can produce up to 10,000 hoverboards in a year. The cost in millions of dollars to produce x thousand hoverboards is given by $C(x) = \frac{1}{2} + \frac{1}{2}x$. The revenue gained from selling x thousand hoverboards is given by $R(x) = -\frac{1}{4}x^2 + \frac{7}{2}x$. What is the maximum profit the company can make in a year?

   1 pt - First note the domain: x is in $[0, 10]$.
   1 pt - Find $P(x) = R(x) - C(x) = -\frac{1}{4}x^2 + 3x - \frac{1}{2}$
   1 pt - Find the only critical point for $P(x)$ (where $P(x) = 0$ or is undefined), $x = 6$.
   2 pt - Use the first or second derivative test to show that $P(x)$ has a local maximum of $-\frac{36}{4} + 18 - \frac{1}{2} = 8.5$ at $x = 2$.
   1 pt - Write a sentence explaining why the test you used leads to the conclusion that $(2, 8.5)$ is a local maximum. 2 pt - Write a sentence explaining that $(2, 8.5)$ is the only critical point and is a local maximum, so it is an absolute maximum.

   You can also get full credit for using the closed interval method.

   More practice for this section:
   (1) What is the maximum profit if the company can only produce 5,000 hoverboards per year?
   (2) If $f(x)$ had more than two critical points, what would you do to find the absolute extrema?

4. Find the following limits. Be sure to show all your work.

   (a) $\lim_{x\to\infty} \left(1 + \frac{5}{x}\right)^{10x}$

   The form of this limit is 1^∞, so it is indeterminate.
   1 pt - Let $L = \lim_{x\to\infty} \left(1 + \frac{5}{x}\right)^{10x}$, find that $\ln(L) = \lim_{x\to\infty} 10x \ln\left(1 + \frac{5}{x}\right)$ Now the form is $\infty \cdot 0$, still indeterminate.
   1 pt - Rewrite as a fraction: $\ln(L) = \lim_{x\to\infty} 10\frac{\ln\left(1 + \frac{5}{x}\right)}{1/x}$. Now the form is $\frac{0}{0}$.
   2 pt - Apply l'Hôpital's rule: $\ln(L) = \lim_{x\to\infty} 10\frac{\frac{1}{1+\frac{5}{x}}\left(\frac{-5}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to\infty} 10\frac{5}{1 + \frac{5}{x}} = 50.$
   2 pt - Conclude that $L = e^{50}$

   (b) $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^5$

   The form of this limit is 1^5 which is not indeterminate.
   6 pts - $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^5 = (1 + 0)^5 = 1^5 = 1.$

Common ways to lose points on these problems:
(1) Dropping limit notation before you actually take the limit.
(2) Attempting to apply l'Hôpital's rule to a limit that doesn't have form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

5. Find the antiderivatives of the following functions.

   (a) $f(x) = \dfrac{3}{x} + \dfrac{5}{x^3}$

      i. $3\ln(x) - \dfrac{5}{2x^2} + c$

      ii. $3\ln|x| - \dfrac{5}{2x^2} + c$

      iii. $-\dfrac{3}{x^2} - \dfrac{5}{x^3} + c$

      iv. $\dfrac{3}{x^2} + \dfrac{5}{x^3} + c$

   (b) $g(t) = \dfrac{3t^4 + 2t^2}{t^2}$

      i. $\dfrac{\frac{3}{5}t^5 + \frac{2}{3}t^3}{\frac{1}{3}t^3} + c$

      ii. $\dfrac{t^2(12t^3 + 4t) - (3t^4 + 2t^2)(4t^3)}{t^4} + c$

      iii. $12t^3 + 4t$

      iv. $t^3 + 2t + c$

   Answer to (a) is ii (5pts). Answer to (b) is iv (5pts).
   Since this is multiple choice, a great strategy if you're unsure is to take the derivatives of the possible answers and see if you get the given equation.

6. The velocity of a comet was measured in kilometers per second at one minute intervals for five minutes. The results of these measurements are given in the table below. What is the approximate change in position of the comet over the five minute period?

Time (min)	0	1	2	3	4	5
Velocity (km/sec)	10	25	35	75	70	71

We can use a left or right hand sum (or find both and take the average). This solution shows a right hand sum.
1 pt - Correctly select endpoints to use: $time = 1, 2, 3, 4, 5$
1 pt - Either convert velocities into km/min or convert times to $60, 120, 180, 240, 300$ seconds
4 pt - Set up the sum: Change in postion$\approx 60v(10) + 60v(120) + 60v(180) + 60v(240) + 60v(300) = 60\,(25 + 35 + 75 + 70 + 71) = 60(276)$ km. It would be fine to leave your answer in this form, but if we want, we can write it as 16,560 km.

Scoring: Total up your points earned and then multiply by 2 to get a score out of 100.