Objectives:

- Begin reviewing for Exam 2. (This is in no way a complete review! See the course website for more review materials.)

Taking Derivatives: Find $\frac{dy}{dx}$ for each of the following:

1. $y = \sin(\arctan(x^2))$

   $y' = \cos(\arctan(x^2))\dfrac{1}{1+x^4}(2x)$

2. $y = 2^x x^2$

   $y' = 2^x(2x) + \ln(2)2^x x^2$

3. $xy = y^3 + 23$

   $xy' + y = 3y^2y'$ so $y' = \dfrac{y}{3y^2 - x}$

4. $y = x^{\sin(x)}$ $//\ln(y) = \sin(x)\ln(x)$ so $\frac{1}{y}y' = \cos(x)\ln(x) + \sin(x)\frac{1}{x}$ and $y' = y\big(\cos(x)\ln(x) + \sin(x)\frac{1}{x}\big) = x^{\sin(x)}\big(\cos(x)\ln(x) + \sin(x)\frac{1}{x}\big)$

5. $y = e^{20}$

   $y' = 0$

6. $y = \sin(x)\csc(x)\tan(x)$

   $y = \tan(x)$ so $y' = \sec^2(x)$

7. $\cos(y)\big(\sin(x)\big)^2 = x^2y^3 + 1$ $//-\sin(y)y'\big(\sin(x)\big)^2 + \cos(y)\big(2\sin(x)\cos(x)\big) = x^2(3y^2)y' + 2xy^3$ so $y' = \dfrac{2\cos(y)\sin(x)\cos(x) - 2xy^3}{3x^2y^2 + \sin(y)\big(\sin(x)\big)^2}$

8. $y = \dfrac{(6x^3-3x)^7}{(x^7-7)^4}$ $//\ln(y) = 7\ln(6x^3-3x) - 4\ln(x^7-7)$ so $y' = y\left(\dfrac{7(18x^2-3)}{6x^3-3x} - \dfrac{4(7x^6)}{x^7-7}\right) = \left(\dfrac{(6x^3-3x)^7}{(x^7-7)^4}\right)\left(\dfrac{7(18x^2-3)}{6x^3-3x} - \dfrac{4(7x^6)}{x^7-7}\right)$

9. $y = e^{\csc(x^3-x)}$

   $y' = e^{\csc(x^3-x)}\big(-\csc(x^3-x)\cot(x^3-x)\big)(3x^2-1)$

10. $y = (3x^2-2x+1)^x$ $//\ \ln(y) = x\ln(3x^2-2x+1)$ so $y' = y\left(\dfrac{6x-2}{3x^2-2x+1}(x) + \ln(3x^2)\right)$

    $y' = (3x^2-2x+1)^x\left(\dfrac{6x-2}{3x^2-2x+1}(x) + \ln(3x^2)\right)$

Applications (Word Problems):

1. Suppose a 5 foot plank is leaning against a wall. The top of the plank is sliding down the wall at a rate of 1 foot per second. How quickly is the bottom of the plank sliding away from the wall when the top of the plank is 4 feet above the ground?

   Let a be the distance from the foot of the plank to the wall, b the distance from the top of the plank to the ground. (Draw a triangle.) We want to find $\dfrac{da}{dt}$ when $b = 4$.

$$a^2 + b^2 = 5^2$$

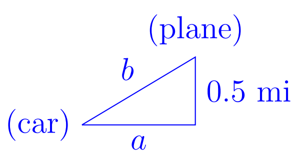
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$2a\frac{da}{dt} + 2(4)(-1) = 0$$

$$\frac{da}{dt} = \frac{8}{2a}$$

   Use pythogorean identity to find that when $b = 4$, $a = 3$. So, $\dfrac{da}{dt} = \dfrac{8}{6} = \dfrac{4}{3}\dfrac{\text{ft}}{\text{sec}}$

2. *Challenge:* In some areas, highway speed limits are enforced by measuring the speed of a car from a plane. Suppose a car is driving on a highway with speed limit 70 mph. A speed enforcement plane flies towards the car at a constant altitude of 0.5 miles and a constant speed of 150 miles per hour. The car drives towards the plane at an unknown speed.
   At a particular time, the pilot's radar indicates that the direct distance from the car to the plane is 1 mile, and this distance is decreasing at a rate of 190 miles per hour. At this point in time, is the car speeding?



   Notice that a=(horizontal position of plane)-(position of car), so $\dfrac{da}{dt}$ = (velocity of plane) - (velocity of car). Thus, velocity of car = (velocity of plane) - $\dfrac{da}{dt} = -150 - \dfrac{da}{dt}$.

$$a^2 + (0.5)^2 = b^2$$

$$2a\frac{da}{dt} + 0 = 2b\frac{dc}{dt}$$

$$\frac{da}{dt} = \frac{b}{a}\frac{db}{dt}$$

   When $b = 1$, we have $a = \sqrt{1 - .25} = \frac{\sqrt{3}}{2}$ and $\frac{db}{dt} = -190$. So, velocity of car $= -150 - \dfrac{da}{dt} = -150 - \frac{1}{\frac{\sqrt{3}}{2}}(-190) = -150 + \frac{380}{\sqrt{3}} \approx 69.4$ mph. The car is (barely) not speeding.