Objectives:

- Review for Exam 1!

Important Note

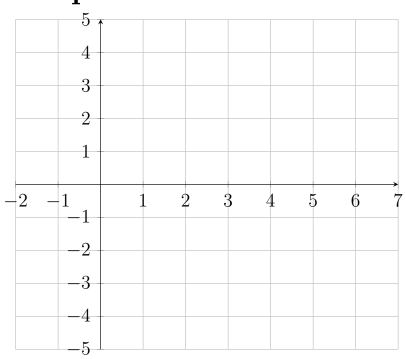
We will focus on the new material covered in Chapter 2. *However,* the precalculus material from sections 1.1-1.3 is fair game for the exam. It would be a good idea to review WebAssign, written homework, and class activities and bring any questions you have to office hours or class on Monday.

This review is meant to supplement, not replace, the review sheet handed out on Monday. Be sure to do those problems as well so you know what to ask questions about on Monday!

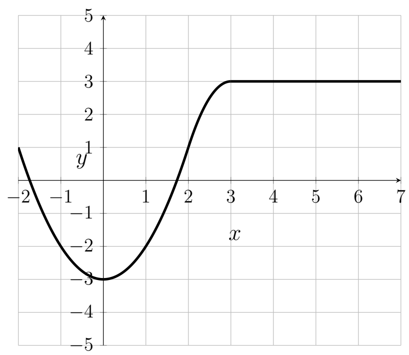
Recap of 2.8

Given a graph of $f'(x)$, we have a good idea of what $f(x)$ will look like.

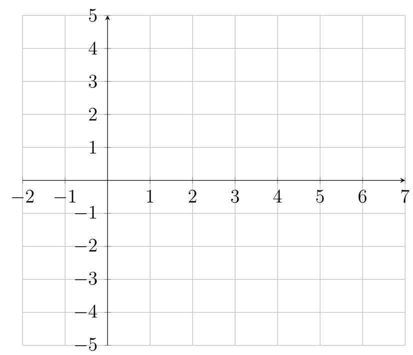
Example



$f(x)$



$f'(x)$

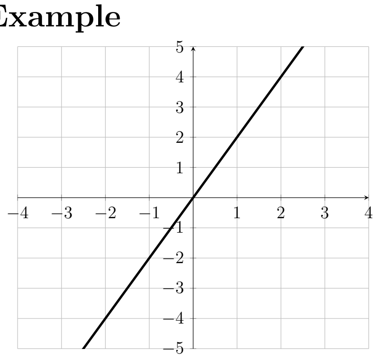


$f''(x)$

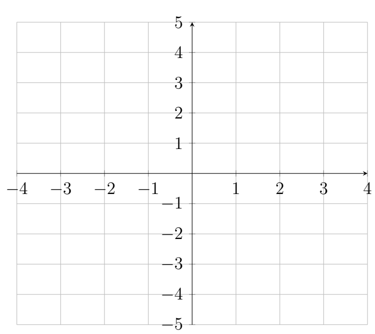
If $f'(x) = g(x)$, $f(x)$ is called an ________________________ of $g(x)$.

$g(x)$ can have more than one antiderivative.

Example



$g(x)$



Some antiderivatives of $g(x)$

Important Definitions and Theorems:

Word Bank

(Some words may be used more than once.)

Continuous	Squeeze	Intermediate Value	Derivative	Removable Discontinuity
x $\quad h$	a $\quad f(a)$	$f(x)$	$f(x+h)$	$f(c)$ $\quad N$
$\lim\limits_{x\to a} f(x)$	$\lim\limits_{x\to a^-} f(x)$	$\lim\limits_{x\to a^+} f(x)$	$\lim\limits_{x\to a} g(x)$	$\lim\limits_{x\to a} h(x)$

$\lim\limits_{x\to a} f(x)$ exists if ________________ = ________________

________________ Theorem: Suppose f is ________________ on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c between a and b such that ________ = ________ .

________________ Theorem: If $h(x)$ ________ $f(x)$ ________ $g(x)$ for all x near a (not necessarily for $x = a$), then ________ $\leq$ ________ $\leq$ ________ .

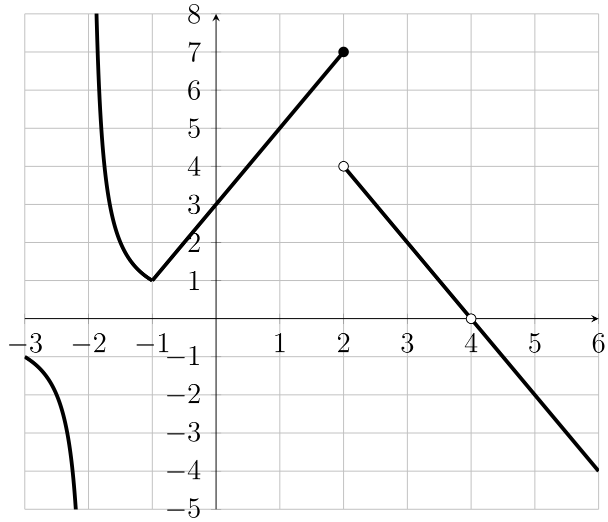
A function is continuous at a if ________________ = ________ .

The derivative of $f(x)$ at $x = a$ can be found in two equivalent ways:

$$f'(a) = \lim_{h\to 0} \frac{\quad - \quad}{\qquad} \qquad \text{or} \qquad f'(a) = \lim_{x\to a} \frac{\quad - \quad}{\quad - \quad}$$

If $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = L$ but $f(a) \neq L$, $f(x)$ has a ________________ at $x = a$.

Continuity and Differentiability



Where is this function, $f(x)$, discontinuous? Specify the type of discontinuity.

Choose some x-value a and use the limit definition of continuity to prove that $f(x)$ is continuous or discontinuous at a.

Where is this function not differentiable?

Application

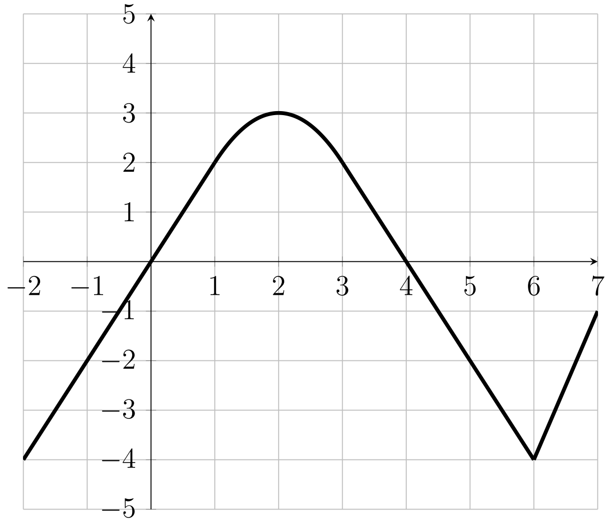
Suppose x is the number of power plants owned by a utilities company and $f(x)$ is the total profit the company makes (in millions of dollars).

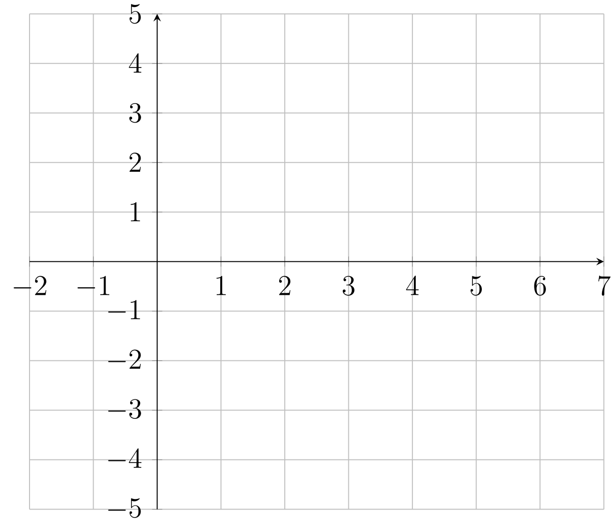
$f(x) = -\dfrac{10}{x} + 3$

(a) Find $f(2)$ and $f(10)$. Include units. What do each of these values represent?

(b) Find $f'(5)$. Include units. What does this value represent?

Below is a graph of the function $f''(x)$. On the blank axes, sketch a graph of $f'''(x)$.





Each of the solution steps below would not receive full credit on the exam. Explain why.

1. $\lim_{x\to 0}\left(\frac{|x|}{x}+4\right)=\lim_{x\to 0}\frac{|x|}{x}+\lim_{x\to 0}4$

2. $\lim_{x\to 3}\frac{x+3}{(x+3)(x+2)}=\frac{1}{x+2}=\frac{1}{5}$

3. *$f(3)=5$ and $f(4)=8$, so $f(c)=7$ for some c in between 3 and 4, by the Intermediate Value Theorem.*

4. $f(x)=x^3$, so $f'(x)=3x^2$.