1. Let y be the solution to the initial-value problem $y' = y$, $y(0) = 1$.

   (a) Use Euler's method with each step size below to estimate the value of $y(0.4)$.

      i. $h = 0.4$

      ii. $h = 0.2$

      iii. $h = 0.1$

   (b) We know that the exact solution of this initial-value problem is $y = e^x$. Draw, as accurately as you can, the graph of $y = e^x$, $0 \leq x \leq 0.4$, together with the Euler approximations using the step sizes in part (a). Use your sketches to decide whether your estimates in part (a) are underestimates or overestimates.

   (c) The error in Euler's method is the difference between the exact value and the approximate value. Find the errors made in part (a) in using Euler's method to estimate the true value of $y(0.4)$, namely $e^{0.4}$. What happens to the error each time the step size is halved?

2. Let $y(x)$ be the solution to the initial-value problem $y' = x + y^2$, $y(0) = 0$.

   (a) Take 1: Use Euler's method with step size 0.2 to estimate $y(0.4)$.

   (b) Take 2: Use Euler's method with step size 0.1 to estimate $y(0.4)$.