Lecture Notes:

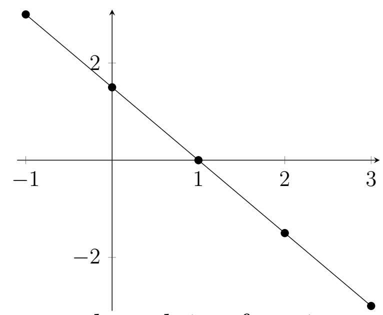
- Parametric equations express both x and y coordinates as a function of a third parameter, often called t. One way to interpret this is to think of t as "time" where $x(t)$ is the x-coordinate of a moving point at time t and $y(t)$ is the y-coordinate of the point at the same time.
  In each of the following examples, draw the start/end points and the direction of motion.

  Example 1:

  – $\begin{cases} x(t) = 1 + 2t \\ y(t) = -3t \end{cases}$
  $-1 \le t \le 1$

  or, $\langle 1 + 2t, -3t \rangle$

t	x(t)	y(t)
-1	-1	3
-1/2	0	3/2
0	1	0
1/2	2	-3/2
1	3	-3



  To graph, we can also eliminate the parameter t by solving for t in terms of one of the other variables and substituting.

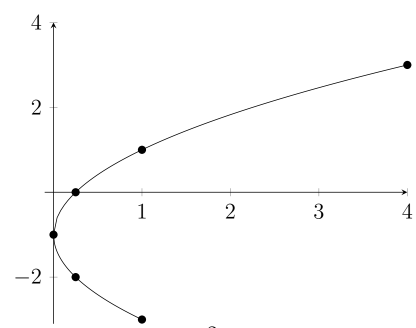
$$\frac{x-1}{2} = t$$
$$y = -3t = -3\left(\frac{x-1}{2}\right) = -\frac{3}{2}x + \frac{3}{2}$$

  Example 2:

  – $\begin{cases} x(t) = t^2 \\ y(t) = 2t - 1 \end{cases}$
  $-1 \le t \le 2$

  or, $\langle t^2, 2t - 1 \rangle$

t	x(t)	y(t)
-1	1	-3
-1/2	1/4	-2
0	0	-1
1/2	1/4	0
1	1	1
3/2	9/4	2
2	4	3

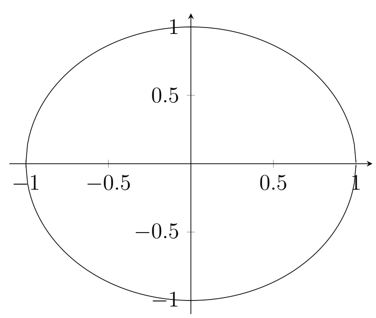


  To eliminate the parameter: $t = \frac{y+1}{2}$, so $x = \left(\frac{y+1}{2}\right)^2$.

  Example 3:

  – $\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \end{cases}$
  $0 \le t \le 2\pi$

  or, $\langle \cos(t), \sin(t) \rangle$



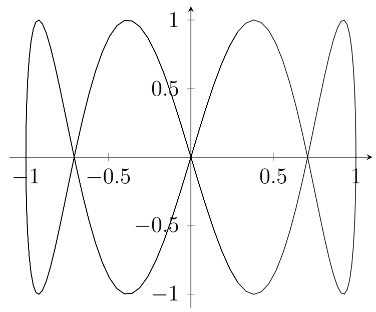
  Something to point out: If we have $\pi \le t \le 3\pi$, we have the same circle but with a different start/end point. If we have $0 \le t \le 4\pi$, we go around the circle twice.

  We can't eliminate the parameter, since arcsin and arccos have limited range, so we won't recover all values of t. (Also this curve is neither a function of x nor a function of y.)

– Example 4:

$$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(4t) \end{cases}$$
$0 \leq t \leq 2\pi$

or, $\langle \cos(t), \sin(4t) \rangle$



Since $\sin(4t)$ completes 4 periods in $0 \leq t \leq 2\pi$ and $\cos(t)$ completes 1 period in $0 \leq t \leq 2\pi$, we see that as the x-values cycle 1, 0, -1, 0, 1, the y values cycle 0, 1, 0, -1, 0 four times.

I might hook my computer up to the projector and show this on Desmos.

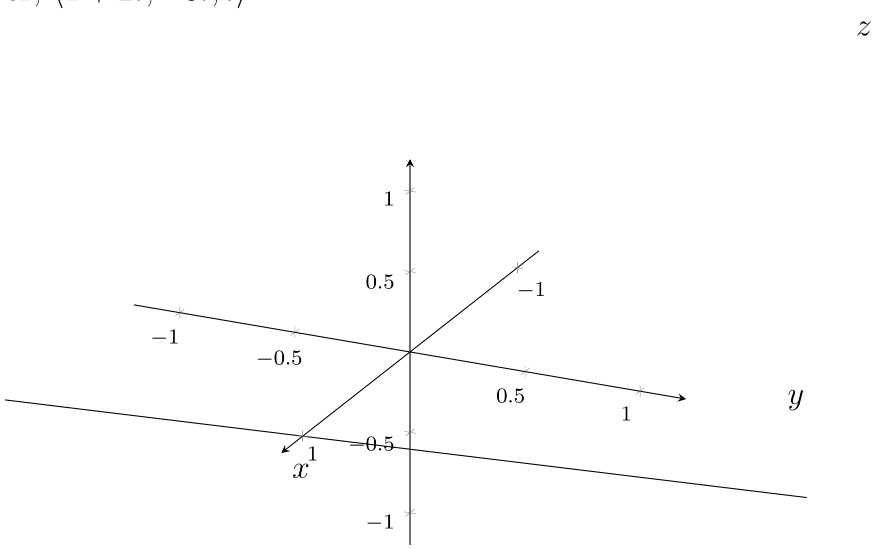
- If we're thinking of parametric equations as the movement of a point over time, we can expand our idea to include movement in 3D space. In order to do this, we need to be able to graph in 3D space!
  – Draw a 3-axis graph and draw some points. (E.G. $(0,0,0)$, $(0,0,2)$, $(0,2,1)$, $(1,0,1)$, $(-1,2,3)$.)

  – Example 1:

$$\begin{cases} x(t) = 1 + 2t \\ y(t) = -3t \\ z(t) = t \end{cases}$$
$-1 \leq t \leq 1$

or, $\langle 1+2t, -3t, t \rangle$

t	x(t)	y(t)	z(t)
-1	-1	3	-1
-1/2	0	3/2	-1/2
0	1	0	0
1/2	2	-3/2	1/2
1	3	-3	1



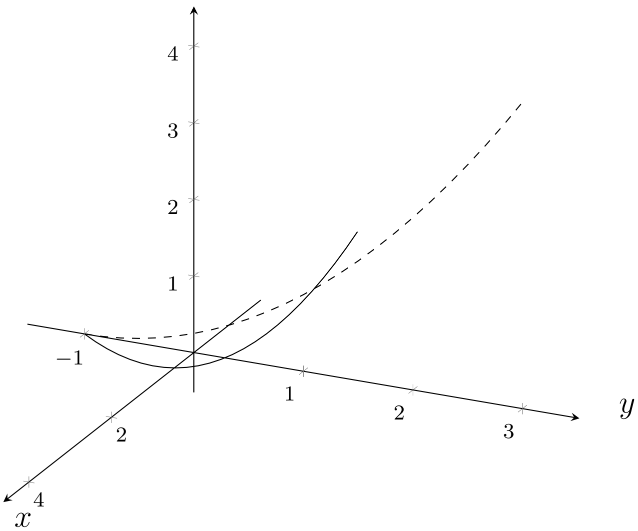
Example 2:

– $\begin{cases} x(t) = 2t \\ y(t) = 2t - 1 \\ z(t) = t^2 \end{cases}$

$0 \leq t \leq 2$

or, $\langle 2t, 2t-1, t^2 \rangle$

t	x(t)	y(t)	z(t)
0	0	-1	0
1/2	1	0	1/4
1	2	1	1
3/2	3	2	9/4
2	4	3	4



This is a good example to talk about traces—they've seen $y(t) = 2t-1, z(t) = t^2$ as $x(t), y(t)$ in Example 2 of the previous section, so they can recognize that trace in the y-z plane.

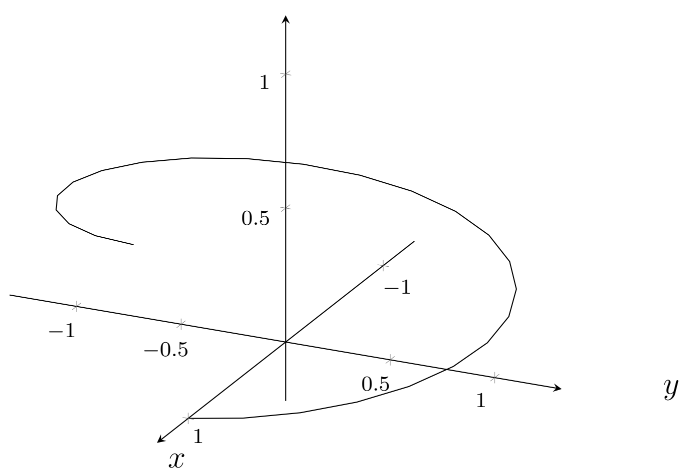
z

Example 3:

– $\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \\ z(t) = t \end{cases}$

$0 \leq t \leq 2\pi$

or, $\langle \cos(t), \sin(t), t \rangle$



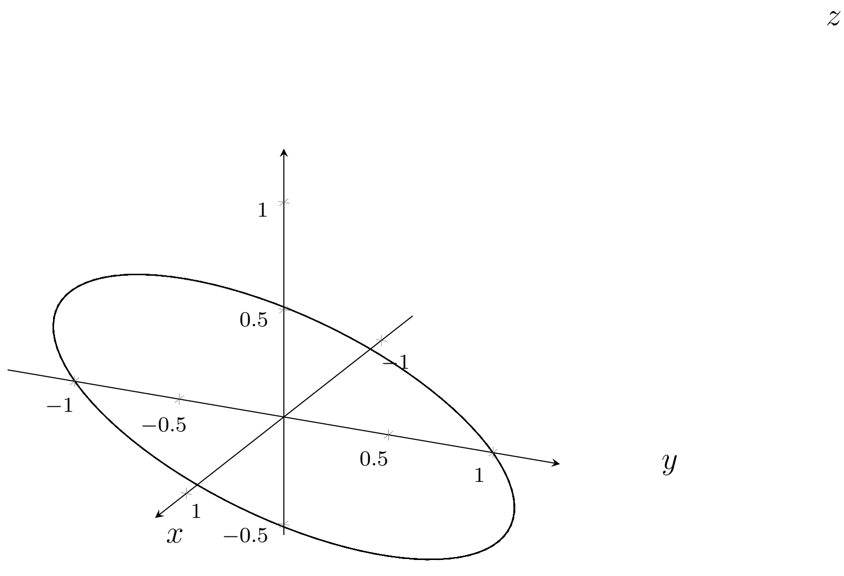
- Example 4:

  $$\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \\ z(t) = \cos(t) \end{cases}$$

  $0 \le t \le 2\pi$

  or,

  $\langle \cos(t), \sin(t), \cos(t) \rangle$



- They can now attempt the matching activity. I would plan on presenting or having someone present the "pringle".