Trig Sub with Completing the Square

Example: Evaluate $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$

$-x^2-2x+3 = -(x^2+2x-3) = -((x+1)^2-4)$

So, $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = \int \frac{x}{\sqrt{4-(x+1)^2}}\,dx$

$$\boxed{\begin{aligned}&\text{Substitute:}\\ &x+1 = 2\sin(\theta)\\ &x = 2\sin(\theta)-1\\ &dx = 2\cos(\theta)\,d\theta\end{aligned}}$$

$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{\sqrt{4-(2\sin(\theta))^2}}\,d\theta$

$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{\sqrt{4(1-(\sin(\theta))^2)}}\,d\theta$

$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{\sqrt{4(\cos(\theta))^2}}\,d\theta$

$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{2(\cos(\theta))}\,d\theta$

$= \int 2\sin(\theta)-1\,d\theta$

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So, $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = \int \frac{x}{\sqrt{4-(x+1)^2}}\,dx$

Substitute:

$$x+1 = 2\sin(\theta)$$
$$x = 2\sin(\theta)-1$$
$$dx = 2\cos(\theta)\,d\theta$$

$$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{\sqrt{4-(2\sin(\theta))^2}}\,d\theta$$

$$= \int \frac{(2\sin(\theta)\ 1)\,2\cos(\theta)}{\sqrt{4\,(1-(\sin(\theta))^2)}}\,d\theta$$

$$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{\sqrt{4\,(\cos(\theta))^2}}\,d\theta$$

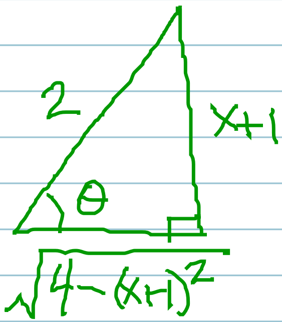
$$= \int \frac{(2\sin(\theta)-1)\,2\cos(\theta)}{2\,(\cos(\theta))}\,d\theta$$

$$= \int 2\sin(\theta)-1\,d\theta$$

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -2\cos(\theta) - \theta + C$$

Remember $\quad x+1 = 2\sin(\theta)$

$$\frac{x+1}{2} = \sin(\theta)$$



$$\cos(\theta) = \frac{a}{h} = \frac{\sqrt{4-(x+1)^2}}{2}$$

Also, $\quad \frac{x+1}{2} = \sin(\theta)$, so $\arcsin\left(\frac{x+1}{2}\right) = \theta$

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -2\left(\frac{\sqrt{4-(x+1)^2}}{2}\right) - \arcsin\left(\frac{x+1}{2}\right) + C$$