Completing the Square with Trig Sub

Example 1: Evaluate $\int \frac{1}{x^2+x+1}\,dx$

$x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4}$

So, $\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

Substitute:
$x+\frac{1}{2} = \sqrt{3/4}\tan(\theta)$
$dx = \sqrt{3}/2 (\sec(\theta))^2 d\theta$

$= \int \frac{\sqrt{3}/2\,(\sec(\theta))^2}{(\sqrt{3}/2\tan(\theta))^2+3/4}\,d\theta$

$= \int \frac{\sqrt{3}/2\,(\sec(\theta))^2}{\frac{3}{4}((\tan(\theta))^2+1)}\,d\theta$

$= \int \frac{\sqrt{3}/2\,(\sec(\theta))^2}{3/4\,(\sec(\theta))^2}\,d\theta$

Solve for θ:
$x+1/2 = \frac{\sqrt{3}}{2}\tan(\theta)$
$\frac{2}{\sqrt{3}}(x+1/2) = \tan(\theta)$
$\arctan\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right) = \theta$

$= \int \frac{\sqrt{3}/2}{3/4}\,d\theta = \int \frac{\sqrt{3}}{2}\cdot\frac{4}{3}\,d\theta$

$= \int \frac{2}{\sqrt{3}}\,d\theta = \frac{2}{\sqrt{3}}\theta + C$

$= \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right)+C$

$= \frac{2\sqrt{3}}{3}\arctan\left(\frac{2\sqrt{3}}{3}(x+\frac{1}{2})\right)+C$

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So, $\int \frac{1}{x^2+x+1}\,dx = \int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$ ($\frac{3}{4}=a^2$)

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