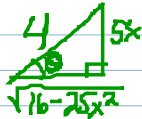


Another Example of Trig Sub.

Example: Evaluate $\int \sqrt{16-25x^2} dx$.Using the substitution below,
we have

Substitute:

$$\sin \theta = \frac{5x}{4}$$

$$\frac{4 \sin \theta}{5} = x$$

$$dx = \frac{4}{5} \cos \theta d\theta$$

$$\int \sqrt{16-25x^2} dx = \int \sqrt{16-25\left(\frac{4 \sin \theta}{5}\right)^2} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int \sqrt{16-25\left(\frac{16 \sin^2 \theta}{25}\right)} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int \sqrt{16-16(\sin^2 \theta)} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int 4\sqrt{1-(\sin^2 \theta)} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \frac{16}{5} \int \sqrt{(\cos^2 \theta)} \cdot \cos \theta d\theta$$

$$= \frac{16}{5} \int \cos \theta \cdot \cos \theta d\theta$$

$$= \frac{16}{5} \int (\cos \theta)^2 d\theta$$

* Power reduction
formula:

$$(\cos \theta)^2 = \frac{1 + \cos(2\theta)}{2}$$

* u-sub:

$$u = 2\theta$$

$$du = 2d\theta$$

$$d\theta = \frac{1}{2} du$$

$$= \frac{16}{5} \int \frac{1 + \cos(2\theta)}{2} d\theta$$

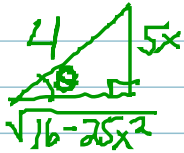
$$= \frac{16}{5} \int \frac{1 + \cos u}{2} \cdot \frac{1}{2} du$$

$$= \frac{16}{5} \cdot \frac{1}{4} \int (1 + \cos u) du$$

$$\frac{4}{5} \cdot (u + \sin u) + C$$

Another Example of Trig Sub.

Example: Evaluate $\int \sqrt{16-25x^2} dx$.



Using the substitution below, we have

Substitute:

$$\sin \theta = 5x/4$$

$$\frac{4 \sin \theta}{5} = x$$

$$dx = \frac{4}{5} \cos \theta d\theta$$

$$\int \sqrt{16-25x^2} dx = \int \sqrt{16-25\left(\frac{4 \sin \theta}{5}\right)^2} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int \sqrt{16-25\left(\frac{16 \sin^2 \theta}{25}\right)} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int \sqrt{16-16 \sin^2 \theta} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \int 4 \sqrt{1-\sin^2 \theta} \cdot \frac{4}{5} \cos \theta d\theta$$

$$= \frac{16}{5} \int \sqrt{\cos^2 \theta} \cdot \cos \theta d\theta$$

$$= \frac{16}{5} \int \cos \theta \cdot \cos \theta d\theta$$

$$= \frac{16}{5} \int (\cos \theta)^2 d\theta$$

$$= \frac{16}{5} \int \frac{1+\cos(2\theta)}{2} d\theta$$

$$= \frac{16}{5} \int \frac{1+\cos u}{2} \cdot \frac{1}{2} du$$

$$= \frac{16}{5} \cdot \frac{1}{4} \int 1+\cos u du$$

$$\frac{4}{5} \cdot (u + \sin u) + C$$

* Power reduction

formula:

$$(\cos \theta)^2 = \frac{1+\cos(2\theta)}{2}$$

* u-sub:

$$u = 2\theta$$

$$du = 2 d\theta$$

$$d\theta = \frac{1}{2} du$$

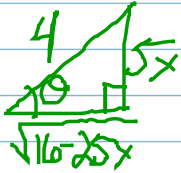
$$\int \sqrt{16-25x^2} dx = \frac{4}{5} (2\theta + \sin(2\theta)) + C$$

* Double angle
formula:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$= \frac{4}{5} (2\theta + 2\sin\theta\cos\theta) + C$$

$$= \frac{4}{5} \left(2\arcsin\left(\frac{5x}{4}\right) + 2 \cdot \frac{5x}{4} \cdot \frac{\sqrt{16-25x^2}}{4} \right) + C$$



$$\sin\theta = \frac{5x}{4}$$

$$\cos\theta = \frac{\sqrt{16-25x^2}}{4}$$

$$\theta = \arcsin\left(\frac{5x}{4}\right)$$