

Objectives:

- State the Fundamental Theorem of Calculus.
- Apply the Fundamental Theorem of Calculus to work with functions defined as integrals.

Example 1 If $g(x) = \int_1^x t^3 dt$, find a formula for $g(x)$ that doesn't involve an integral and calculate $g'(x)$.

We can compute $g(x)$ with the Evaluation Theorem:

$$g(x) = \int_1^x t^3 dt = \left. \frac{t^4}{4} \right|_1^x = \frac{x^4 - 1}{4}.$$

Now, we can take the derivative directly:

$$g'(x) = \frac{d}{dx} \left(\frac{1}{4}x^4 - \frac{1}{4} \right) = x^3$$

Weird! The derivative of $g(x) = \int_1^x t^3 dt$ is eerily similar to the integrand, t^3 .

The Fundamental Theorem of Calculus (Part I)

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t) dt \quad a \leq x \leq b$$

is an antiderivative of f , that is, $g'(x) = f(x)$ for $a < x < b$.

Example 2 Find the derivative of the function $g(x) = \int_0^x \sqrt{1+t^2} dt$.

Since $f(t) = \sqrt{1+t^2}$ is continuous for $x \geq 0$, we can use Part I of the Fundamental Theorem of Calculus:

$$g'(x) = \sqrt{1+x^2}.$$

The Fundamental Theorem of Calculus

Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t) dt$, then $g'(x) = f(x)$.

2. $\int_a^b f(x) dx = F(b) - F(a)$, where F is any antiderivative of f , that is $F' = f$.

For each of the following functions, find the derivative.

1. $g(x) = \int_1^x \frac{1}{t^3 + 1} dt$

4. $f(x) = \int_1^{x^4} \sec(t) dt$

2. $r(y) = \int_2^y t^2 \sin(t) dt$

5. $h(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1} du$

3. $F(x) = \int_x^\pi \sqrt{1 + \sec(t)} dt$