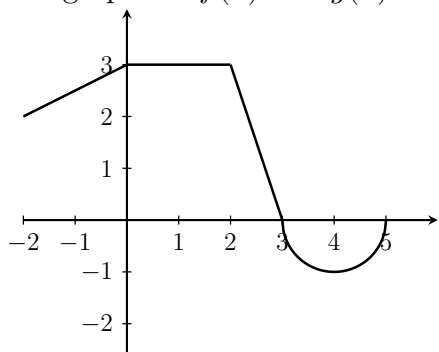


Objectives:

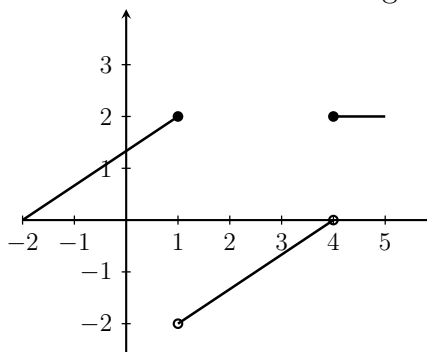
- Practice using properties of definite integrals.
- Compare values of definite integrals.
- Use antiderivatives to evaluate definite integrals.

1. $\int_0^{2\pi} (x + \sin(x)) dx$

2. The graphs of $f(x)$ and $g(x)$ are given below. Calculate the integrals.



$f(x)$



$g(x)$

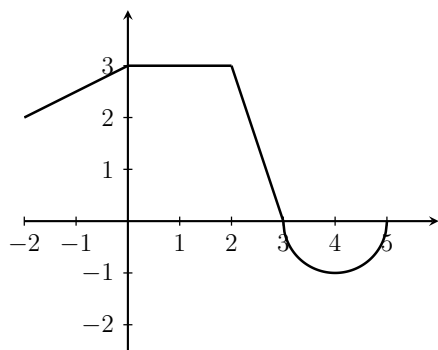
(a) $\int_{-2}^1 f(x) + g(x) dx$

(b) $\int_2^5 10f(x) dx =$

(c) $\int_{-2}^1 g(x) + 5 dx$

(d) $\int_2^1 g(x) dx + \int_4^2 g(x) dx$

Comparing Integrals: For the function $f(x)$ in the previous problem, draw a function $h(x)$ on the axis such that $h(x) \geq f(x)$ for all x values in the interval $[0, 5]$:



How does $\int_0^5 h(x)$ compare to $\int_0^5 f(x)$?

In general we can say that if $f(x) \leq h(x)$ for all x in the interval $[a, b]$, then $\int_a^b f(x) \leq \int_a^b h(x)$.

In particular:

(1) If $f(x) \geq 0$ for all x in $[a, b]$:

(2) If $m \leq f(x) \leq M$ for all x in $[a, b]$ where m, M are constants:

Example: It would be very difficult to calculate $\int_{-2}^3 \sin\left(\frac{1}{x}\right) dx$. However, we can compare the integral we want to know about to integrals that are easy to compute:

Evaluation Theorem (or, Fundamental Theorem of Calculus, Part II)

If f is _____ on _____

and F is any _____, (i.e. $F'(x) = \text{_____}$), then

We use the notation _____ to denote _____.

Note:

Examples

1. $\int_{-1}^2 x^4 dx$

2. $\int_0^1 \frac{1}{1+x^2} dx$

3. $\int_2^{10} \left(e^x + 5x - \frac{1}{x} \right) dx$

Because of this relationship between the integral of $f(x)$ and the antiderivative of $f(x)$, we write _____ to mean _____. We call this expression an _____.

Note:

So now we have 3 ways of calculating an indefinite integral:

Interpreting the integral:

The Evaluation Theorem also appears as the _____.

Since $F'(x) = f(x)$ is the _____, the Evaluation Theorem tell us that the _____ is equal to _____, which we call the _____.

Examples:

If $f(x)$ represents:	Then $\int_a^b f(x)dx = F(b) - F(a)$ represents:
Velocity	
Marginal Cost	
Growth Rate of a Population	

Note: