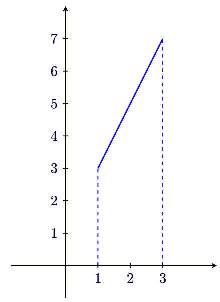
Objectives:

- Estimate areas under curves.
- Estimate the distance traveled by an object using velocity.

Example 1 How can we compute the area between the x-axis and the line $2x+1$ between $x=1$ and $x=3$?

There's more than one way to do this, but we are going to need to know how to compute the area of a trapezoid in the future, so let's practice using that:

$$\text{Area of a trapezoid} = \left(\frac{\text{base 1} + \text{base 2}}{2}\right)\text{height} = \left(\frac{3+7}{2}\right)(2) = 10$$

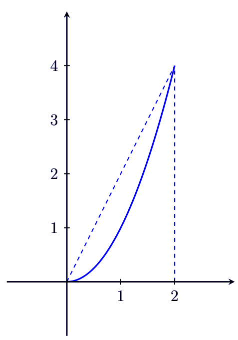


Example 2 How can we compute the area between the x-axis and the curve x^2 between $x=0$ and $x=2$?

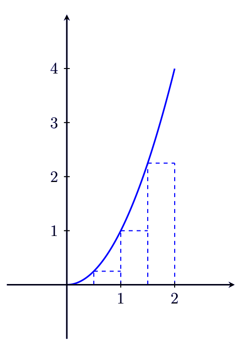
We can estimate using shapes we understand such as ___triangles, rectangles, or trapezoids___.

Triangle:

Area$=\dfrac{2\cdot 4}{2} = 4$

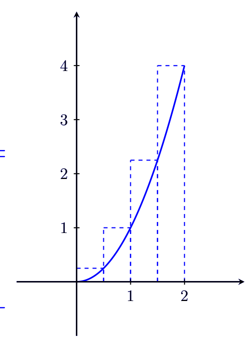


Left-Riemann sum
$n=4$ rectangles
widths: $\Delta x = 1/2$
Endpoints: $x = 0, 1/2, 1, 3/2$
Heights: $f(0) = 0, f(1/2) = 1/4, f(1) = 1, f(3/2) = 9/4$
Left-hand sum:
Area$=(1/2)(0)+(1/2)(1/4)+(1/2)(9/4)+(1/2)(1)=7/4$



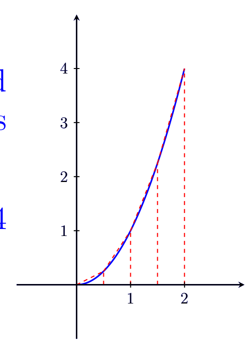
Right-Riemann sum
$n=4$ rectangles
widths: $\Delta x = 1/2$
Endpoints: $x = 1/2, 1, 3/2, 2$
Heights: $f(1/2) = 1/4, f(1) = 1, f(3/2) = 9/4, f(2) = 4$

Right-hand sum:
Area$=(1/2)(1/4) + (1/2)(1) + (1/2)(9/4) + (1/2)(4) = 15/4$



How can we make a more accurate estimate?
We can average the right-hand and left-hand sums. This gives the trapezoidal estimate.

$$\text{Area} = \frac{(15/4)+(7/4)}{2} = 11/4$$



To make any of these estimates more accurate, ___increase value of n___.

Note: We don't have to use the left or right endpoints of an interval to determine the height of each rectangle. We could come up with different endpoints by using:
the height at the middle of the interval, the largest or smallest height in the interval, the height 7/8th of the way across the interval, or the height at a random point in each interval

Example 3 Estimate the area between the x-axis and the curve $10 - 2x$ between $x = 0$ and $x = 3$.

Let's use $n = 3$ rectangles.

Right-hand estimate:

$\Delta x = \dfrac{3-0}{3} = 1$ so our endpoints are $x = 1, 2, 3$ with heights $f(1) = 8$, $f(2) = 6$, $f(3) = 4$, so the right hand estimate is $1(8) + 1(6) + 1(4) = 18$

Left-hand estimate:

$\Delta x = \dfrac{3-0}{3} = 1$ so our endpoints are $x = 0, 1, 2$ with heights $f(0) = 10$, $f(1) = 8$, $f(2) = 6$, so the left hand estimate is $1(10) + 1(8) + 1(6) = 24$

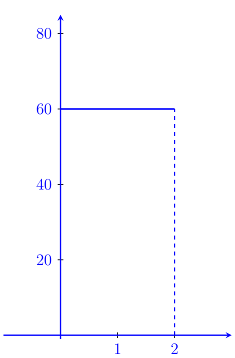
Trapezoidal estimate:

To find the trapezoidal estimate, we average the left and right estimates: $\dfrac{18+24}{2} = 21$

Exact value:

Notice that this is a linear function so the area under the function IS a trapezoid. So the exact area is $\left(\dfrac{10+4}{2}\right)3 = 21$. Since the area is a trapezoid, this is exactly the same as our trapezoidal estimate above. For most curves this will not be the case!!

Example 4 Velocity and distance Recall that distance = rate × time. For example, if you travel 60 mph for 2 hours, you will have travled $60\dfrac{\text{mi}}{\text{hr}}2$ hr = 120 mi. If we look at a graph of the velocity, we can see this is equal to the area of the rectangle formed between $v(t)$ and the x-axis between $t = 0$ and $t = 2$:



If $v(t)$ varies depending on t, the area between $v(t)$ and the horizontal axis will still be equal to the distance traveled. This area can be estimated using Riemann sums.

For example, use the chart of velocity values below to find the total distance traveled by an object between $t = 0$ and $t = 6$:

t (min)	0	2	4	6	8
$v(t)$ (ft/min)	5	8	7	11	0

We can compute a couple of different estimates:

- 4 rectangles, left hand estimate: $2(5) + 2(8) + 2(7) + 2(11) = 62$ feet
- 2 rectangles, right hand estimate: $4(7) + 4(0) = 14$ feet
- 2 rectangles, midpoint estimate: $4(8) + 4(11) = 76$ feet
- 4 rectangles, underestimate: $2(5) + 2(7) + 2(7) + 2(0) = 38$ feet
- 4 rectangles, overestimate: $2(8) + 2(8) + 2(11) + 2(11) = 54$ feet