Objectives:

- Define an antiderivative.
- Compute the antiderivatives of some familiar functions.

Let $f(x)$ be any function. We call $g(x)$ an antiderivative of $f(x)$ if $g'(x) = f(x)$.

For example, if $f(x) = 3$, some antiderivatives of $f(x)$ are: $3x,\ 3x+10,\ 3x-10,000$

A way we could represent all of these solutions is: $3x + c$ where c is any constant.

In general, we write the antiderviative of $f(x)$ as $F(x) + c$ where $F'(x) = f(x)$.

$f(x)$	Antiderivative of $f(x)$
$f(x) = 0$	$F(x) = c$
$f(x) = 5$	$F(x) = 5x + c$
$f(x) = 3x^2$	$F(x) = x^3$
$f(x) = x^2$	$F(x) = \frac{1}{3}x^3 + c$
$f(x) = x$	$F(x) = \frac{1}{2}x^2 + c$
$f(x) = x^n$	$F(x) = \frac{1}{n+1}x^{n+1} + c$
$f(x) = \frac{1}{x}$	$F(x) = \ln(x) + c$
$f(x) = 5x^2$	$F(x) = \frac{5}{3}x^2 + c$
$f(x) = x - 3$	$F(x) = \frac{1}{2}x^2 - 3x + c$
$f(x) = e^x$	$F(x) = e^x + c + c$
$f(x) = b^x$	$F(x) = \frac{b^x}{\ln(b)} + c$

Some Antiderivative Rules:
If the antiderivative of $f(x)$ is $F(x)+c$ and the antiderivative of $g(x)$ is $G(x)+c$ then the antiderivative of $f(x)+g(x)$ is

$$F(x)+G(x)+c$$

If the antiderivative of $f(x)$ is $F(x)+c$ and b is a constant, the antiderviative of $b \cdot f(x)$ is

$$b \cdot F(x)+c$$

Examples: Find the antiderivatives, (Don't forget "$+c$"!):

1. $F'(x) = 3x^4 + 7x^2 + 5$

   $f(x) = \frac{3}{5}x^5 + \frac{7}{3}x^2 + 5x + c$

2. $G'(z) = \dfrac{z^2+1}{\sqrt{z}}$

   $G'(z) = z^{3/2} + z^{-1/2}$ so $G(z) = \frac{2}{5}z^{5/2} - 2z^{1/2} + c$

3. $k'(t) = \dfrac{2}{3} + \dfrac{4}{t} + \dfrac{7}{\sqrt{t}}$

   $k'(t) = \frac{2}{3} + 4t^{-1} + 7t^{-1/2}$ so $k(t) = \dfrac{2}{3}t + 4\ln(t) + 14t^{1/2} + c$

More antiderivatives!

$f(x)$	Antiderivative of $f(x)$
$f(x) = \cos(x)$	$F(x) = \sin(x) + c$
$f(x) = \sin(x)$	$F(x) = -\cos(x) + c$
$f(x) = (\sec(x))^2$	$F(x) = \tan(x) + c$
$f(x) = \sec(x)\tan(x)$	$F(x) = \sec(x) + c$
$f(x) = \frac{1}{1+x^2}$	$F(x) = \arctan(x) + c$
$f(x) = \frac{1}{\sqrt{1-x^2}}$	$F(x) = \arcsin(x) + c$

More Examples: Find the antiderivatives:

1. $H'(x) = \sin(x) + \pi + (\sec(x))^2$

   $H(x) = -\cos(x) + \pi x + \tan(x) + c$

2. $s'(t) = 2^x - \cos(x)$

   $s(t) = \frac{1}{\ln(2)} 2^x - \sin(x) + c$

(Did you remember to include "$+c$"?)

Initial value problems: Given $f'(x)$, we have seen that we can find $f(x) + c$. If we also know the value of $f(x)$ at some point, we can find the value of the constant c.

1. $s'(t) = -32t + 8$ and $s(0) = 40$. Find an equation for $s(t)$.

   $s(t) = -16t^2 + 8t + c$. Now use $s(0) = 40$. $40 = -16(0)^2 + 8(0) + c$, so $40 = c$. So we know $s(t) = -16t^2 + 8t + 40$.

2. $f''(\theta) = \sin(\theta) + \cos(\theta)$ and $f'(0) = 3, f(0) = 4$. (Find $f(\theta)$.)

   $f'(\theta) = -\cos(\theta) + \sin(\theta) + c$.

   So $3 = -\cos(0) + \sin(0) + c = -1 + c$ and so $c = 4$.

   Then $f'(\theta) = -\cos(\theta) + \sin(\theta) + 4$, so $f(\theta) = -\sin(\theta) - \cos(\theta) + 4\theta + b$.

   Now $4 = -\sin(0) - \cos(0) + 4(0) + b = -1 + b$ so $b = 5$.

   Then $f(\theta) = -\sin(\theta) - \cos(\theta) + 4\theta + 5$

3. A stopped car accelerated at $4\frac{\text{m}}{\text{sec}^2}$ for 6 sec. Find a formula for velocity, $v(t)$, and a formula for position, $s(t)$.

   Since the car started from a stationary position, we know $v(0) = 0$ and $s(0) = 0$.
   Acceleration is the derivative of velocity, so to find $v(t)$ we take the antiderivative of $a(t) = 4$. Then $v(t) = 4t + c$. Use initial condition to solve for c: $0 = 4(0) + c$ so $c = 0$. Then $v(t) = 4t$. Velocity is the derivative of position, so to find $s(t)$ we take the antiderivative of $v(t)$. Then $s(t) = 2t^2 + b$. Use initial condition: $0 = 2(0)^2 + b$ leads to $b = 0$. So $s(t) = 2t^2$.

4. Acceleration due to gravity on earth is $-32\frac{\text{ft}}{\text{sec}^2}$. A pumpkin is dropped from a 64 ft tall building. How long does it take to hit the ground and what is the impact velocity?

   $v(0) = 0$, $s(0) = 64$
   $v(t) = -32t$,
   $s(t) = -16t^2 + 64 = -16(t^2 - 4) = -16(t + 2)(t - 2)$ so the pumpkin hits the ground at $t = 2$sec.
   $v(2) = -32(2) = -64$ so the impact velocity is $-64\frac{\text{ft}}{\text{sec}}$