Objectives:

- Find a function that models a problem and apply the techniques from 4.1, 4.2, and 4.3 the find the optimal or "best" value.

Suggested procedure:

Step 1. Draw a picture! Label variables and known quantities.

Step 2. Decide what quantity we want to maximize (or minimize).

Step 3. Find a formula for the quantity that we want to maximize (or minimize).

Step 4. Use constraints to turn our formula into an equation in one variable.

Step 5. Find the domain.

Step 6. Find the global minimum (or maximum).

(a) If we are looking in a closed interval: substitute endpoints and critical points into the function and choose the largest (or smallest) value.

(b) If we are looking in an open interval: Hope there is only one critical point, show there is a local maximum (or minimum) there, conclude it is also a global maximum (or minimum).

Step 7. Remember to answer the original question clearly and completely!

Example 1. Find two non-negative numbers whose sum is 200 and whose product is maximum.

Example 2. The corners are cut our of an $8\frac{1}{2}'' \times 11''$ piece of paper and it is folded into a box. What size squares should be removed to maximize the volume?

Example 3. A rectangle is inscribed in the triangle with vertices $(0,0)$, $(4,0)$, and $(0,8)$ with one side of the rectangle on lying on the x-axis and one side of the rectangle lying on the y-axis. What is the maximum area of the rectangle?

Example 4. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Example 5. A rectangular mural will have a total area of 24 ft^2 which includes a border of 1 ft on the left, right, and bottom and a border of 2 ft on the top. What dimensions maximize the total paintable area inside the borders.

Example 6. A can is made to hold 1 liter of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.
(Note: 1 liter is equivalent to 1,000 cubic centimeters.)

Example 7. A glass fish tank is to be constructed to hold 72 ft^3 of water. The top is to be open. The width will be 5 ft but the length and the depth are variable. Building the tank costs \$10 per square foot for the base and \$5 per square foot for the sides. What is the cost of the least expensive tank?

Example 8. A baseball team plays in a stadium that holds $55,000$ spectators. with ticket prices at \$10, the average attendance has been $27,000$. Some financial experts estimated that prices should be determined by the function $p(x) = 19 - \frac{1}{3000}x$ where x is the number of tickets sold. What should the price per ticket be to maximize revenue?