Objectives:

- Review indeterminate forms of limits.
- Introduce and practice using l'Hospital's Rule, a derivative based tool for evaluating limits.

Recap of Indeterminate Forms

Earlier in the semester, we encountered indeterminate forms while taking limits.

For example, $\lim_{x\to\infty} \frac{x^4 - 3x^2 + 4}{x^2 + 1}$ is an indeterminate form, because:

Note: We don't write ______________________________________ .

Instead write ____ .

Types of Indeterminate Forms:

Forms that are <u>not</u> indeterminate:

In the past we have dealt with indeterminate forms by attempting to rewrite the existing function in a way that doesn't lead to an indeterminate form.

Examples of various methods:

Find $\lim_{x\to\infty} \frac{x^4 - 3x^2 + 4}{x^2 + 1}$ if it exists.

Find $\lim_{x\to\infty} \sqrt{x^2 + 3x} - x$ if it exists.

However, there are some limits where this doesn't work. For example, $\lim_{x \to \infty} \frac{x^2}{e^x}$ has the indeterminate form $\frac{\infty}{\infty}$ but we can't simplify it with any of our existing strategies.

Now that we know how to take derivatives, we can add another tool to our indeterminate form toolbox:

l'Hospital's Rule: (also written l'Hôpital's Rule)

If $f(x)$ and $g(x)$ are ____________________ and $\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form __________

or __________, then _________________________

l'Hospital's rule applies for:

Why?

Examples:

1. $\lim_{x \to \infty} \frac{x+3}{x^2+2x}$

2. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

3. $\lim_{x \to \infty} \frac{x^2}{e^x}$

4. $\lim_{x \to 0} \frac{x}{\cos(x)}$

5. $\lim_{x \to 0} \frac{x}{\sin(x)}$