Objectives:

- Define and use the Mean Value Theorem.

Suppose you drive from Boulder to Denver along I-25 and US36. You decide to drive in the toll lane for 23 miles. Two weeks later you receive a speeding ticket in the mail. The ticket claims that you entered the toll road at 4:45 pm and exited 23 miles later at 5:00 pm. If you decide to appeal the ticket, can they prove you were speeding?

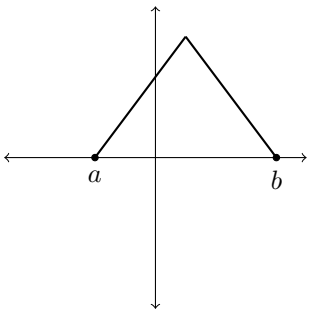
The Mean Value Theorem (MVT):

If f is _____ on _____, and f is _____ on _____, then there exists _____ such that

In words, this means that if f meets these requirements over an interval $[a, b]$, then there is a point in the interval where the _____ at that point is equal to the _____ of the interval.

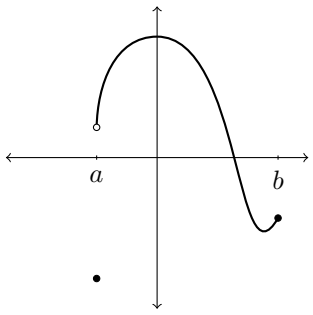
Graphically, this means that if f satisfies these hypotheses over $[a, b]$, then some point in the interval has a _____ that is _____ to _____

For the following functions, are the hypotheses of the Mean Value Theorem true? Is the conclusion of the Mean Value Theorem true?



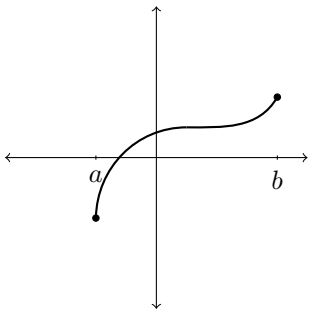
Hypotheses: Not differentiable on (a, b)

Conlcusion: No. Secant line slope= 0. No point has horizontal tangent.



Hypotheses: Not continuous on $[a, b]$

Conclusion: True! (Draw secant and tangent lines)



Hypotheses: True

Conclusion: Must be true by MVT. (There are even 2 such pts)

Back to our example: We could represent the position of the car, $s(t)$ in miles, as a function of time, t in minutes after 4:45pm.

Does this function on the interval _____ meet the hypotheses of the Mean Value Theorem?

What does the Mean Value Theorem imply in this case?

In general, how does the Mean Value Theorem apply to velocity?

More Examples: What does the MVT tell us about the following functions?

1. $f(x) = x^2$ on $[1, 3]$

2. $g(t) = \dfrac{1}{t}$ on $[-1, 1]$

Example: Prove $f(x) = x^3 + x + 1$ has only one zero.