

**Objectives:**

- Find local maxima and minima of a function graphically.
- Find the absolute maximum and the absolute minimum of a function graphically.
- Use the first derivative test to find local extrema analytically.

**Definitions:**

- **CRITICAL NUMBER:** A number  $c$  in the domain of a function  $f$  is a **critical number** of  $f$  if  $f'(c) = 0$  or  $f'(c)$  does not exist.

- **ABSOLUTE MAXIMUM VALUE:** For a number  $c$  in the domain of a function  $f$ , the number  $f(c)$  is the **absolute maximum value** of  $f(x)$  if

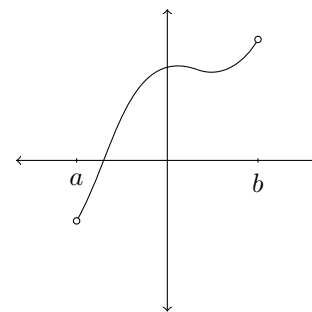
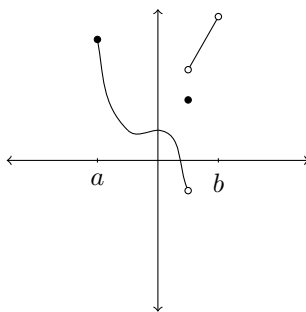
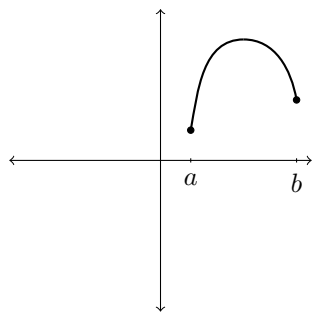
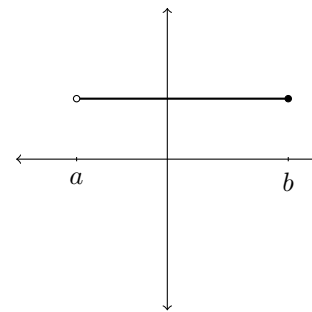
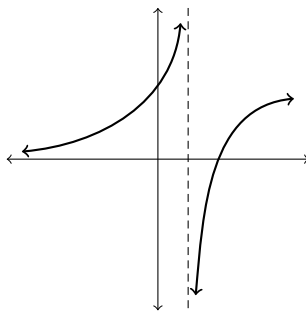
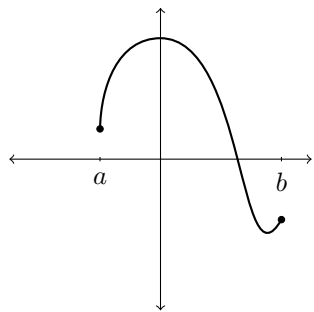
$$f(c) \geq f(x) \text{ for all } x \text{ in the domain of } f.$$

- **ABSOLUTE MINIMUM VALUE:** For a number  $c$  in the domain of a function  $f$ , the number  $f(c)$  is the **absolute minimum value** of  $f(x)$  if

$$f(c) \leq f(x) \text{ for all } x \text{ in the domain of } f.$$

We refer to absolute maximum and minimum values as absolute extrema.

Functions don't always have absolute extrema. Which of these functions have absolute extrema?



You will work more with absolute extrema in your project tomorrow!

**Definitions:**

- LOCAL MAXIMUM VALUE: The number  $f(c)$  is a **local maximum value** of  $f(x)$  if

$$f(c) \geq f(x) \text{ when } x \text{ is near } c.$$

- LOCAL MINIMUM VALUE: The number  $f(c)$  is a **local minimum value** of  $f(x)$  if

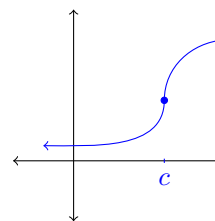
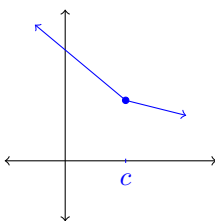
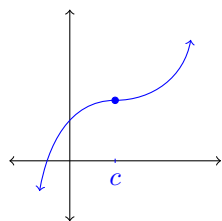
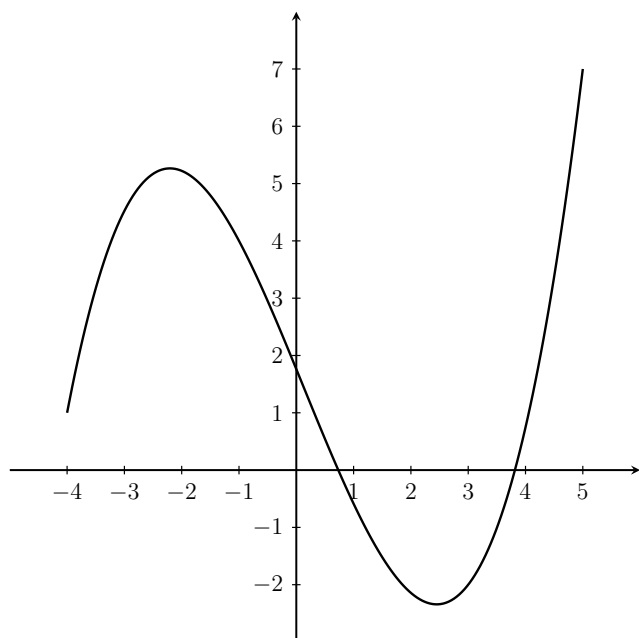
$$f(c) \leq f(x) \text{ when } x \text{ is near } c.$$

We refer to local maximums and minimums as local extrema.  
If we are on a hunt for local extrema, where should we look?

**Fermat's Theorem:**

If  $f$  has a local maximum or minimum at  $c$ , then either  $f'(c) = 0$  or  $f'(c)$  does not exist (i.e.,  $c$  is a critical number).

CAUTION: the converse is not true! If  $f$  has a critical number  $c$ ,  $f$  doesn't necessarily have a local extreme value there:

**Graphical example:**

- $f$  is defined on  $[-4, 5]$ .
- At  $x = 5$ ,  $f$  has an absolute max of 7.
- At  $x = 2.5$ ,  $f$  has a local min of about  $-2.25$  (also an absolute min).
- At  $x = -2.5$ ,  $f$  has a local max of 5 (not an absolute max).

So, if  $c$  is a critical number for  $f$ , how do we tell if  $f$  has a local max, local min, or neither?

Analytically, there are two options (described in Section 4.3). Here is the first one:

**First Derivative Test:** If  $c$  is a critical number for a continuous function  $f$ ,

- (a) If  $f'(x)$  changes from positive to negative at  $c$ , then  $f$  has a local max at  $x = c$  .
- (b) If  $f'(x)$  changes from negative to positive at  $c$ , then  $f$  has a local min at  $x = c$  .
- (c) If  $f'(x)$  does not switch signs, then  $f$  has no local extremum at  $x = c$  .

**Example:** Find and classify all critical numbers of

$$f(x) = 2x^3 + 3x^2 - 12x.$$

Steps:

1. Find  $f'(x)$ .

$$f'(x) = 6x^2 + 6x - 12$$

2. Find where  $f'(x) = 0$  or where  $f'(x)$  DNE.

$$0 = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x - 1)(x + 2)$$

So  $x = 1$  and  $x = -2$  are critical numbers.

There are no other critical numbers because  $f'(x)$  is always defined.

3. Make a sign chart for  $f'(x)$ .

$$f'(x) \quad \leftarrow \begin{array}{ccc} + & - & + \\ | & | & \\ -2 & & 1 \end{array} \rightarrow$$

4. Use the first derivative test to classify each critical number.

At  $x = -2$ , the derivative switches from positive to negative so the first derivative test tells us that  $f(x)$  has a local max at  $x = -2$ . The local maximum value is  $f(-2) = 20$ .

At  $x = 1$ , the derivative switches from negative to positive so the first derivative test tells us that  $f(x)$  has a local min at  $x = 1$ . The local minimum value is  $f(1) = -7$ .