Objectives:

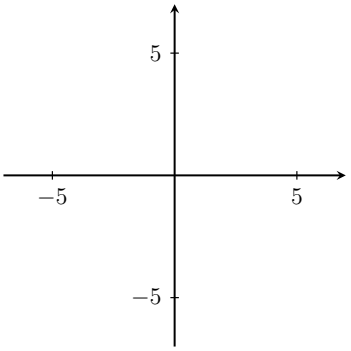
- Find derivatives of implicit functions.

Background:

If we have a formula involving x and y, like

$$x^2 + y^2 = 25,$$

we have a curve that essentially defines y as a function of x near a specific point even though it isn't solved for y. We say y is implicitly a function of x.

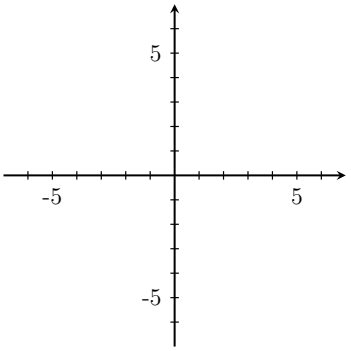


Main Idea: We can sneakily find $\frac{dy}{dx}$ (in other words, y') without solving explicitly for y.

How? Differentiate both sides of the equation remembering all the while that y is a function of x.

Example 1 Consider $x^2 + y^2 = 25$. Find the slope of the tangent line at the point $(3, 4)$.

Step 1. Differentiate:



Step 2. Solve for y':

Step 3. Substitute values:

Further questions:

1. What is the equation of the tangent line at the point $(3, 4)$?

2. Where is the tangent line horizontal?

3. Where is the tangent line vertical?

Example 2 Find a formula for y' and find where the line tangent to the curve is vertical for the curve given by

$$x^2 + xy + x + y = 1$$

Differentiate:

Solve for y':

Example 3 Find the equation of the tangent line to the curve given below at the point $(1, 2)$.

$$x^3 + y^3 + x^2y^2 = 13$$