Objectives:

- Find the derivative of functions that are the product or quotient of other functions.

The Product Rule
If f, g are both differentiable functions:

$$\frac{d}{dx}\left(f(x)g(x)\right) =$$

Like all our derivative rules, we can prove the Product Rule using the limit definition of a derivative. If you'd like to prove it on your own, here's a hint:

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h} = \lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

Examples

(a) $f(x) = x^{11}5^x$. Find $f'(x)$.

(b) $g(t) = \left(6t^5 - 3t + 1\right)e^t$. Find the derivative of $g(t)$.

Key Example: Using the Product Rule more than once. $h(t) = e^t 9^t t^6$. Find $\frac{d}{dt}h(t)$

The Quotient Rule
If f, g are both differentiable functions:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$$

Note:

Later in the course we will have the tools to prove the Quotient Rule follows from the Product Rule.

For example: We can find the derivative of $h(t) = \dfrac{2^x}{x^3}$ using the quotient rule or the product rule.

What about the derivative of $g(x) = \dfrac{x^3}{2^x} = x^3 2^{-x}$?

More Examples

(a) $f(x) = \dfrac{(3x-1)2^x}{x^3-1}$

(b) Last time we only proved the product rule for positive integer exponents.
Use the Quotient Rule to show that $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$ for any positive integer n.

(c) $h(x) = f(x)g(x)$ where $f(x) = 4^x$ and the values of g, g' are given by

x	0	1	2
$g(x)$	2	5	11
$g'(x)$	3	7	19

Find $h'(0), h'(1)$, and $h(2)$.