Objectives:

- Define relationships between $f(x)$, $f'(x)$ and $f''(x)$.
- Use information from $f(x)$ to graph $f'(x)$.

What does $f(x)$ tell us about $f'(x)$?

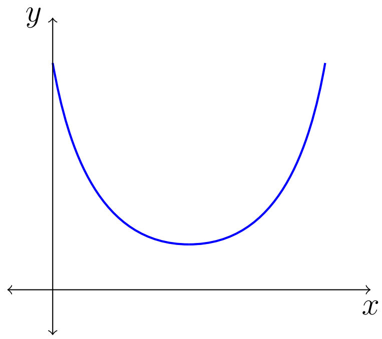
If $f(x)$ is increasing at $x = a$, then $f'(a)$ is positive.

If $f(x)$ is decreasing at $x = a$, then $f'(a)$ is negative.

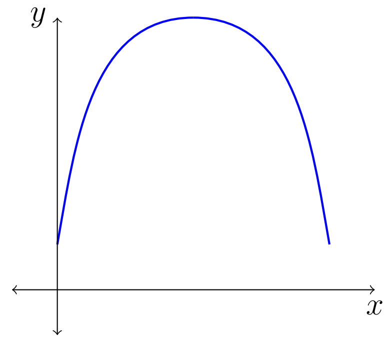
If $f(x)$ has a horizontal tangent line at $x = a$, then $f'(a) =$ 0.

Note: If $f(x)$ is discontinuous at a, has a corner/cusp at a, or has a vertical tangent line at a, then $f'(a)$ is undefined.

What does $f(x)$ tell us about $f''(x)$?



Concave up



Concave down

If $f(x)$ is concave up, $f'(x)$ is increasing, so $f''(x)$ is positive.

If $f(x)$ is concave down, $f'(x)$ is decreasing, so $f''(x)$ is negative.

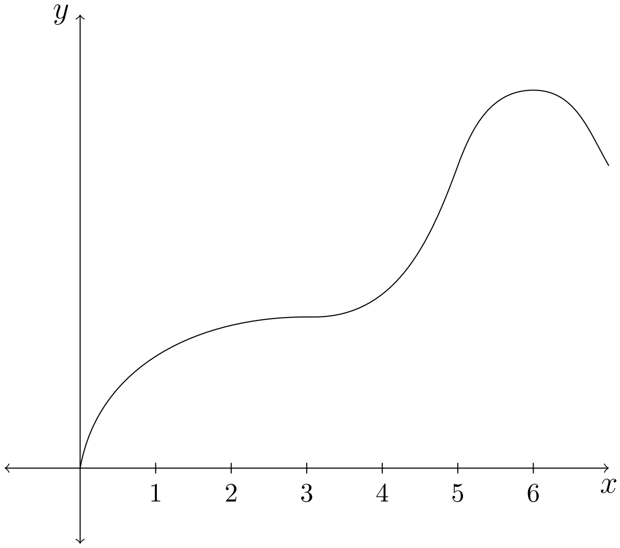
Summary

First, look for points where the derivative or second derivative is zero. Then consider where $f'(x)$ and $f''(x)$ are positive or negative, according to the following patterns:

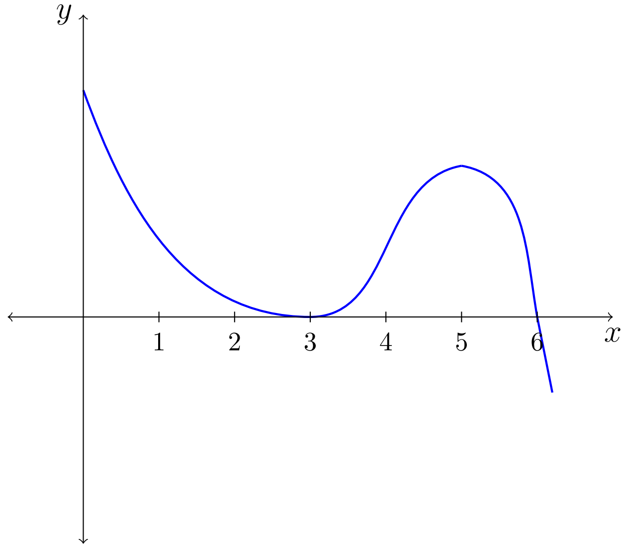
$f(x)$	increasing	decreasing
$f'(x)$	+	−

$f(x)$	concave up	concave up
$f'(x)$	increasing	decreasing
$f''(x)$	+	−

Example:



$f(x)$



$f'(x)$

	$x \in (1,3)$	$x = 3$	$x \in (3,6)$	$x = 6$	$x \in (6,7)$
$f(x)$	increasing	horizontal tangent	increasing	horizontal tangent	decreasing
$f'(x)$	+	0	+	0	−

	$x \in (1,3)$	$x \in (3,5)$	$x \in (5,7)$
$f(x)$	concave down	concave up	concave down
$f'(x)$	decreasing	increasing	decreasing
$f''(x)$	−	0	0

We'll use these basic rules in today's class activity. The solutions to the activity will be posted on the course website - I would recommend adding at least some of those examples to your notes.